VOLUME LXXI NUMBER 5

JUL 14 1930

THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

Edited by

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BEHAVIOR OF BRIGHT LINES IN THE SPECTRA OF SEVERAL LONG-PERIOD VARIABLE STARS - - - - - - *Paul W. Merrill and Cora G. Burwell* 285

A SPECTROGRAPHIC STUDY OF U SAGITTAE - - - - - - *Alfred H. Joy* 336

THE RAYTON SHORT-FOCUS SPECTROGRAPHIC OBJECTIVE - *Milton L. Humason* 351

INDEX - 357

THE UNIVERSITY OF CHICAGO PRESS
CHICAGO, ILLINOIS, U.S.A.

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For the British Empire, except North America, India, and Australasia: The Cambridge University Press, Fetter Lane, London, E.C. 4. Yearly subscriptions, including postage, £1 9*s*. 3*d*. each; single copies, including postage, 3*s*. 9*d*. each.

For Japan: The Maruzen Company, Ltd., Tokyo.

For China: The Commercial Press, Ltd., Paoshan Road, Shanghai. Yearly subscriptions, $6.00; single copies, 75 cents, or their equivalents in Chinese money. Postage extra, on yearly subscriptions 50 cents, on single copies 5 cents.

Claims for missing numbers should be made within the month following the regular month of publication. The publishers expect to supply missing numbers free only when losses have been sustained in transit, and when the reserve stock will permit.

Business Correspondence should be addressed to The University of Chicago Press, Chicago, Illinois.

Communications for the editors and manuscripts should be addressed to the Editors of THE ASTROPHYSICAL JOURNAL, Yerkes Observatory, Williams Bay, Wisconsin.

The cable address is "Observatory, Williamsbay, Wisconsin."

The articles in this Journal are indexed in the *International Index to Periodicals*, New York, N.Y.

Entered as second-class matter, January 17, 1895, at the Post-Office at Chicago, Ill., under the act of March 3, 1879.
Acceptance for mailing at special rate of postage provided for in Section 1103, Act of October 3, 1917, authorized on July 25, 1918.

PRINTED IN THE U.S.A.

THE ASTROPHYSICAL JOURNAL

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VOLUME LXXI JUNE 1930 NUMBER 5

BEHAVIOR OF BRIGHT LINES IN THE SPECTRA OF SEVERAL LONG-PERIOD VARIABLE STARS[1]

By PAUL W. MERRILL and CORA G. BURWELL

ABSTRACT

The *variables investigated* include ten stars of type Me and three of type Se (see Table I). Numerous spectrograms are listed in Table II.

Intensities of bright lines.—Estimated intensities of *Hδ*, *Hγ*, *Hβ*, *Fe* 4202, *Fe* 4308, and *Mg* 4571 served for the study of the behavior of these lines during the light-cycle. The relative intensities of pairs of lines rather than the intensities of individual lines were employed, thus avoiding serious photometric difficulties. The *intensity ratio* *Hγ*:*Hδ* in Me stars varies with phase in a striking manner. Typical behavior is as follows: At one-fifth of the period before maximum, phase −0.2, the ratio is 1/8; it rises rapidly to 1/2 at −0.05, then remains nearly constant until +0.2, after which it again rises rapidly to 2½ at about +0.4. In Se spectra the ratio has a minimum of 2½ at phase +0.07, with higher values earlier and later. The *intensity ratio* *Hβ*:*Hγ* is not well de fined in Me stars because *Hβ* is seldom observed, and its intensity seems to vary in different cycles. In general, *Hβ* grows relatively stronger after maximum. In Se spectra *Hβ* is very intense. The ratio to *Hγ* is high at early and late phases, with a minimum after maximum light. The *intensity ratio* λ 4308:λ 4202 behaves as follows in Me spectra: It is about 1/4 at phase +0.2, and increases steadily to 1 at +0.3 and 4 at +0.55. In type Se, λ 4308 is 2½ times as intense as λ 4202 whenever both are visible. In Me spectra the *intensity ratio* λ 4571:λ 4308 has the following typical values: 2/3 at phase +0.15, 1 at +0.30, and 3 at +0.6.

Wave-lengths and identifications of lines.—The mean wave-lengths of fifty-eight bright lines are recorded in Table VIII. Most of these are true emission lines although a few may be maxima in the absorption spectrum. The elements identified include *H*, *Fe* I, *Fe* II (both allowed and forbidden lines), *Mg* I, *Si* I, *Mn* I, *Sr* II?, *In* I?. The forbidden iron lines and an ultra-violet magnesium triplet have not previously been recorded in these spectra. Data on individual stars, including wave-length measurements and the phases at which certain lines appear, are given in Tables V, VI, and VII and in the notes to Table VIII.

Displacements of bright lines.—The apparent velocities from the *blue-violet lines* are given in Table II. Plots showing the variation with phase are given for eleven stars in Figs. 16, 17, and 18. Algebraically decreasing velocity at and shortly after light-maximum, and a flat minimum 30–80 days after light-maximum are typical features. A

[1] *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 399.

maximum in the displacement-curve about 30 days before light-maximum is probably also typical. The range in apparent velocity is about 10 km/sec., probably being larger for the stars with the shorter periods. The phenomena appear essentially the same for Me and Se variables. The mean displacement of the *bright Hα line* relative to the blue-violet bright lines is found to be −0.34 A for three stars of type Me and −0.04 A for two of type Se.

Discussion.—Most of the bright lines behave as if they had little *dependence on the photosphere.* In discussing the *effects of band absorption,* the existence of titanium oxide above the level at which the bright lines are produced is suggested as a working hypothesis to explain the abnormally low intensities of *Hα*, *Hβ*, and *Hγ* relative to *Hδ*. The fraction of the total band absorption to which the bright lines are subject may be greatest at the first appearance of the lines after minimum and grow less as the phase advances. The bearing of the behavior of the bright lines on *theories of long-period variables* is briefly discussed.

INTRODUCTION

Bright lines are well-known features of the spectra of numerous long-period variable stars, the most conspicuous being the hydrogen lines *Hδ*, *Hγ*, and sometimes *Hβ*; the iron lines λλ 4202, 4308; and the magnesium line λ 4571. Many instances of variations in intensities of these lines, especially those of hydrogen, have been noted by the Harvard observers.[1] The behavior of the bright lines in the spectrum of *o* Ceti has been studied in detail by Stebbins[2] and by Joy;[3] and one of the present writers has found that approximately the same course of variations is exhibited by several other M-type variables.[4] Observations of the same lines in S-type variables have revealed systematic differences as compared with the Me variables.[5] The principal object of the present paper is a more detailed intercomparison than has heretofore been made of the behavior of the bright lines in several variables.

Photometric data[6] concerning the objects studied are collected in Table I. The next to the last column, $M-m$, gives the interval in days between the minimum and the following maximum of the mean light-curve. The last column gives the fraction obtained by dividing this interval by the period.

The present *Contribution* deals chiefly with the intensities and

[1] See *Harvard Annals,* **56,** 209, 1912; *Popular Astronomy,* **36,** 229, 1928; and elsewhere.

[2] *Lick Observatory Bulletins,* **2,** 78, 1903; *Astrophysical Journal,* **18,** 341, 1903.

[3] *Mt. Wilson Contr.,* No. 311; *Astrophysical Journal,* **63,** 281, 1926.

[4] Merrill, *Mt. Wilson Contr.,* No. 200; *Astrophysical Journal,* **53,** 185, 1921. *Mt. Wilson Contr.,* No. 264; *Astrophysical Journal,* **58,** 215, 1923.

[5] Merrill, *Mt. Wilson Contr.,* No. 325; *Astrophysical Journal,* **65,** 23, 1927.

[6] *Harvard Annals,* **79,** 161, 1928.

displacements of the bright lines in the blue-violet region of the spectrum. Outstanding bright lines, other than *Hα*, apparently do not occur in the green, yellow, or red. Even *Hα*, except in the S-type variables, is much less intense than *Hγ* or *Hδ*. This line has been observed occasionally, however, and a few measurements of its displacement are recorded.

The photographs listed in Table II were obtained with one-prism spectrographs attached to the 100-inch telescope (C series) or the 60-inch telescope (γ series). Most of them were taken with an 18-

TABLE I

PHOTOMETRIC DATA CONCERNING VARIABLES STUDIED

Star	R.A. 1900	Dec. 1900	Spect.	Max. Mag.	Range in Mags.	Period in Days	$M-m$ in Days	$\frac{M-m}{p}$
T Cassiopeiae	0^h17^m8	$+55°\ 14'$	M8e	8.2	3.7	443	247	0.56
o Ceti	2 14.3	− 3 26	M6e	3.4	5.8	331.6	120	.36
U Orionis	5 49.9	+20 10	M8e	6.0	5.8	372.2	131	.35
R Geminorum	7 1.3	+22 52	Se	7.1	6.1	370	131	.35
R Leonis	9 42.2	+11 54	M8e	5.9	4.2	313	136	.43
S Ursae Majoris	12 39.6	+61 38	Se	7.9	3.6	225.3	110	.49
R Hydrae	13 24.2	−22 46	M7e	4.2	5.3	405	197	.49
W Hydrae	13 43.4	−27 52	M8e	7.1	2.8	385.6	198	.51
R Serpentis	15 46.1	+15 26	M7e	6.9	6.1	357.2	140	.39
R Cygni	19 34.1	+49 58	Se	7.4	6.5	425.5	157	.37
χ Cygni	19 46.7	+32 40	M6pe	5.1	8.2	406.6	167	.41
T Cephei	21 8.2	+68 5	M7e	6.1	4.0	387	211	.54
R Cassiopeiae	23 53.3	+50 50	M7e	7.1	5.8	431.4	167	0.39

inch camera; a number taken with a 10-inch camera are indicated by an asterisk, while a few with a 7-inch camera are indicated by a double asterisk. The photometric data were supplied through the kindness of Shapley and Leon Campbell, of the Harvard College Observatory.

The numerical values of the intensities of *Hβ*, *Hγ*, *Hδ*, λλ 4202, 4308, and 4571, given in Table II, are estimates made directly from the plates as viewed in the measuring microscope. The scale was established by assuming that intensity 1 corresponds to a line just strong enough to allow an accurate setting of the micrometer wire. Other intensities represent the observer's attempt to estimate the amount of darkened silver grains in the image in terms of the standard line of intensity 1. Intensities 2–4 correspond to lines best

TABLE II

JOURNAL OF OBSERVATIONS AND DATA CONCERNING BRIGHT LINES

PLATE	DATE	MAG.	PHASE DAYS	VEL. KM/SEC.	INTENSITIES					
					$H\delta$	4202	4308	$H\gamma$	4571	$H\beta$
T Cassiopeiae										
C 21	1919 Aug. 29	9.1	−132	−25.9	3	.....	.....	0.5	.....	.....
25	Aug. 31	9.0	−130	−25.9	2	.....	.....	.....	.....	.....
48	Sept. 5	8.9	−125	.....	4	.....	.....	0.5	.....	.....
55	Sept. 7	8.8	−123	−22.0	1	.....	.....	.....	.....	.....
γ 8724	Oct. 3	8.2	− 97	−26.8	1	.....	.....	.....	.....	.....
C 124	Oct. 14	8.1	− 86	−20.8	3	0.6	.....	1	.....	.....
192	Nov. 9	8.3	− 60	−25.4	4	.9	.....	1	.....	.....
2359*	1923 July 26	8.4	− 39	−22.5	6	.....	.....	2	.....	.....
2413*	Aug. 30	7.9	− 4	−26.0	5	.6	.....	2	.....	.....
2471	Oct. 1	7.8	+ 28	−27.7	2	0.6	.....	2	.....	.....
2513*	Oct. 27	8.0	+ 54	−23.2	2	1	0.8	2	.....	.....
γ 13163	1925 Jan. 5	7.7	+ 56	−22.3	3	0.9	.4	4	.....	.....
15949	1928 July 5	8.1	− 35	−26.8	3	.....	.....	0.8	.....	.....
16053	Aug. 4	7.7	− 5	−28.1	4	.6	.....	2	.....	.....
16133	Aug. 30	7.6	+ 21	−21.3	4	0.8	.2	3	.....	.....
C 5023	Oct. 4	8.4	+ 56	−22.0	2	1	0.9	2	0.8	0.5
U Orionis										
C 44	1919 Sept. 4	8.0	+ 75	−32.7	8	0.8	.....	4	.....	.....
638	1920 Sept. 2	7.0	+ 46	−40.0	10	0.9	.....	7	.....	.....
671	Sept. 26	8.0	+ 70	−39.3	10	2	.....	8	.....	.....
727	Oct. 26	8.6	+100	−35.2	8	2	0.8	6	0.9	0.2
2472	1923 Oct. 1	7.4	+ 44	−33.8	10	0.8	.....	7	.....	.....
2518	Oct. 28	8.1	+ 71	−33.9	9	1	0.2	5	.....	.....
2651*	1924 Jan. 23	10.3	+158	−31.1	2	3	4	1	7	0.2
2704*	Feb. 22	11.0	+188	−23	.....	0.2	0.4	.....	0.9	.....
3989.5*	1926 Aug. 23	6.7	− 23	.....	2	.....	.....	0.8	.....	.....
3990*	Aug. 23	6.7	− 23	−29	6	.....	.....	3	.....	.....
γ 14467	Aug. 25	6.6	− 21	−32.7	4	.....	.....	3	.....	.....
C 4015	Sept. 17	5.9	+ 2	−37.2	8	.....	.....	5	.....	1
γ 14532	Sept. 20	5.9	+ 5	−36.9	8	.....	.....	7	.....	1
C 4067	Oct. 22	6.6	+ 37	−39.2	8	0.8	.....	5	.....	0.5
γ 14751	Nov. 27	8.4	+ 73	−43.5	7	1	.4	4	0.5	0.4
C 4481	1927 Oct. 13	6.8	+ 21	−30.3	12	.....	.....	7	.....	.....
4488*a*	Oct. 14	6.9	+ 22	−32.5	7	.....	.....	6	.....	.....
4488*b*	Oct. 14	6.9	+ 22	−34.5	7	.....	.....	5	.....	.....
γ 15352	Oct. 19	7.2	+ 27	−36.9	4	.....	.....	2	.....	.....
C 4504*b*	Oct. 28	7.5	+ 36	−37.9	0.5	.....	.....	.....	.....	.....
4504*c*	Oct. 28	7.5	+ 36	−39.1	2	.....	.....	0.6	.....	.....
4504*d*	Oct. 28	7.5	+ 36	−37.8	2	.....	.....	0.8	.....	.....
4538	Nov. 10	7.8	+ 49	−32.4	10	0.6	.....	6	.....	.....
4611	1928 Jan. 4	8.9	+104	−32.6	6	0.9	.....	4	.....	.....
4677*	Feb. 10	(9.8)	+141	−31.4	3	1	0.4	3	.....	.....

TABLE II—*Continued*

	Plate	Date	Mag.	Phase Days	Vel. Km/Sec.	Intensities					
						$H\delta$	4202	4308	$H\gamma$	4571	$H\beta$
					R Geminorum						
γ	8727	1919 Oct. 3	7.4	− 23	−57.8	0.4	.....	.....	3	.....	8
C	127	Oct. 14	7.0	− 12	−54.3	2	.....	.....	6	.....	10
	133	Oct. 15	7.0	− 11	−55.5	2	.....	.....	6	.....	12
	144	Oct. 17	7.0	− 9	−56.5	2	.....	.....	7	.....	12
	238	1920 Jan. 9	8.7	+ 75	.....	.....	.....	.....	0.2	.....	0.4
	681	Sept. 28	7.2	− 30	−52.3	.....	.....	.....	2	.....	8
	821	Dec. 27	8.3	+ 60	−59.8	0.4	.....	Tr	4	.....	9
	1339	1921 Sept. 21	8.1	− 50	−57.5	.....	.....	.....	0.4	.....	4
γ	10524	Oct. 10	7.2	− 31	−62.2	0.9	.....	.....	6	.....	14
C	1402	Oct. 13	7.1	− 28	−52.8	.....	.....	.....	(1)	.....	6
	1408	Oct. 14	7.1	− 27	−52.2	.....	.....	.....	?	.....	4
	1456	Nov. 13	7.2	+ 3	−58.7	2	.....	Tr?	8	.....	15
	1501	Dec. 15	7.3	+ 35	−55.2	2	.....	0.4	7	.....	12
	1502	Dec. 15	7.3	+ 35	−54.9	0.2	.....	0.2	2	.....	8
	2003	1922 Nov. 6	6.6	+ 10	−56.0	6	0.4	.....	14	.....	(25)
	2062	1923 Jan. 6	7.6	+ 71	−62.0	1	.....	Tr?	5	.....	9
	2480*	Oct. 3	9.1	− 58	−60.1	.....	.....	.....	1	.....	4
	2515*	Oct. 27	7.8	− 34	.....	.....	.....	.....	2	.....	6
	2519	Oct. 28	7.8	− 33	−52.9	.....	.....	.....	1	.....	7
	2563*	Nov. 27	7.3	− 3	−58.4	.....	.....	.....	2	.....	8
	2701*	1924 Feb. 21	9.2	+ 83	−53.5	.....	.....	0.4	5	.....	12
γ	13098	Nov. 11	6.8	− 9	−63.4	2	.....	.....	9	.....	18
C	3097	Dec. 13	7.0	+ 23	−58.5	6	.4	.7	12	.....	25
	3574*	1925 Oct. 30	6.8	− 20	−57.5	2	.....	.....	9	.....	20
	3623	Dec. 26	7.1	+ 37	−55.5	4	.2	.8	10	.....	24
	3647	1926 Jan. 4	7.2	+ 46	−58.0	2	.....	0.3	7	.....	13
	3671*	Jan. 25	7.7	+ 67	−52.6	4	0.6	1	9	.....	16
	3694*	Feb. 21	9.0	+ 94	−56.0	3	1	2	8	0.6	15
					R Leonis						
C	257	1920 Jan. 16	9.0	− 64	+ 3.1	2	.....	.....	.....	.....	.....
	367	Apr. 8	6.4	+ 19	− 1.6	14	2	.....	10	.....	.....
γ	9147	Apr. 11	6.5	+ 22	− 4.5	14	2	.....	10	.....	.....
	9148	Apr. 11	6.5	+ 22	− 4.9	9	0.8	.....	5	.....	.....
	9173	May 5	7.4	+ 46	− 1.6	14	4	2	13	1	.....
	9201	May 11	7.6	+ 52	− 1.0	13	4	3	14	2	.....
C	446	May 30	8.3	+ 71	+ 1.6	4	3	2	5	2	.....
	818	Dec. 26	5.8	− 15	− 0.3	10	.....	.....	6	.....	.....
	865	1921 Jan. 28	6.2	+ 18	− 1.6	15	1	.....	10	.....	.....
	910	Feb. 25	6.9	+ 46	+ 0.8	15	4	.....	10	.....	.....
	918	Feb. 26	6.9	+ 47	+ 1.0	16	4	1	10	.....	.....
	960	Mar. 26	7.7	+ 75	− 2.0	6	2	1	4	1	.....
	963	Mar. 27	7.7	+ 76	+ 2.6	9	4	2	6	2	.....
	1004	Apr. 28	8.7	+108	+ 1.7	2	4	4	3	5	.....
	1504	Dec. 15	6.0	+ 12	− 5.8	10	1	.....	6	.....	.....
	2016	1922 Dec. 8	7.0	+ 63	+ 2.4	4	0.8	.....	2	.....	.....
	2564*	1923 Nov. 27	9.0	+117	− 0.8	0.8	4	7	2	10	.....

TABLE II—*Continued*

Plate	Date	Mag.	Phase Days	Vel. Km/Sec.	Intensities					
					Hδ	4202	4308	*Hγ*	4571	*Hβ*
R Leonis—*Continued*										
2569	1923 Nov. 28	9.0	+118	− 0.3	.....	(0.3)	0.8	.....	1	.....
2594	Dec. 24	9.7	+144	+ 0.9	.....	1	3	.....	5	.....
2652*	1924 Jan. 23	10.2	+174	+ 2	.....	1	3	Tr	6	.....
2798*	May 21	6.9	− 36	+ 3.8	10	.....	.....	2	.....	.....
2802	May 22	6.8	− 35	+ 2.1	3	.....	.....	0.8	.....	.....
γ 14763	1926 Dec. 14	6.1	− 8	+ 0.8	8	.....	.....	4	.....	.....
C 4192	1927 Feb. 18	6.6	+ 58	− 3.6	5	0.8	.....	2	.....	.....
4226*	Apr. 14	8.2	+113	− 1.0	2	4	4	2	4	.....
4267*	May 13	8.8	+142	− 8.2	.....	1	2	.....	3	.....
4273*	May 15	8.8	+144	− 1.6	.....	2	3	.....	4	.....
4593	Dec. 23	6.6	+ 50	− 8.0	10	3	0.9	7	0.6	.....
4612	1928 Jan. 4	7.0	+ 62	− 6.0	8	3	2	8	1	.....
4616*a*	Jan. 5	7.1	+ 63	−12.0	3	0.9	0.4	2	.....	.....
4616*b*	Jan. 5	7.1	+ 63	−12.1	2	Tr	.....	0.9	.....	.....
4678*	Feb. 10	8.3	+ 99	+ 0.7	6	5	6	4	5	.....
4681	Feb. 11	8.3	+100	− 2.7	6	6	7	5	8	.....
4707*	Mar. 16	8.9	+134	+ 9.5	1	4	6	1	10	0.7
4712	Mar. 17	8.9	+135	+ 6.4	0.2	2	3	0.4	5	.....
4745*	Apr. 12	9.7	+161	+11.3	.....	0.6	2	.....	3	.....
4748*	Apr. 14	9.8	+163	− 7.5	.....	5	8	0.9	14	.....
4793*	May 2	10.0	+181	+ 7.7	.....	0.2	1	.....	2	.....
4796*	May 3	10.0	+182	+ 3.0	.....	0.9	2	.....	5	.....
5088*	Nov. 29	7.4	+ 65	+ 3.9	7	2	1	4	1	.....
5098	Dec. 20	8.6	+ 86	+ 0.4	2	1	1	1	2	Tr
5129*	1929 Jan. 24	8.9	+121	+ 2.9	0.6	4	6	0.7	9	.....
5141*	Feb. 28	10.2	+156	+11.6	.....	0.7	2	.....	3	.....
5152*	Mar. 20	10.2	+176	+ 5.4	.....	(0.3)	0.6	.....	1	.....
5172*	Apr. 26	9.7	− 92	.....	.....	.....	.....	.....	.....	.....
5188*	May 22	8.6	− 66	.....	Tr	.....	.....	.....	.....	.....
5191*	May 23	8.6	− 65	+ 7	0.6	.....	.....	.....	.....	.....
5219	June 21	7.0	− 36	+ 5	1	.....	.....	0.5	.....	.....
S Ursae Majoris										
C 369	1920 Apr. 8	8.5	− 37	− 4.2	.....	.....	.....	2	.....	6
373	Apr. 10	8.5	− 35	− 7.2	.....	.....	.....	4	.....	9
γ 9252	June 3	8.2	+ 19	− 6.6	.....	.....	.....	2	.....	5
9848	Dec. 29	8.4	+ 18	−10.1	0.6	.....	.....	3	.....	4
C 866	1921 Jan. 28	8.7	+ 48	− 7.1	.....	.....	Tr	2	.....	4
1169**	July 23	8.1	− 3	−13.4	.8	.....	.....	4	.....	8
1553	1922 Feb. 11	8.5	− 26	+ 4.1	.....	.....	.....	3	.....	7
1626	Mar. 19	7.6	+ 10	− 3.7	.8	.....	.....	5	.....	7
γ 11744**	1923 May 1	8.5	− 42	+ 6.2	.....	.....	.....	4	.....	10
C 2286	June 4	8.0	− 8	− 0.6	0.8	.....	.....	4	.....	10
2340	July 3	8.1	+ 21	− 6.8	2	.....	Tr	6	.....	10
2648	1924 Jan. 22	8.0	− 19	− 5.8	0.8	.....	.....	6	.....	11
2702*	Feb. 21	8.2	+ 11	− 1.1	0.7	.....	.....	5	.....	8
3187*	1925 Mar. 3	8.5	− 55	+ 9.6	.....	.....	.....	4	.....	8

TABLE II—*Continued*

Plate	Date	Mag.	Phase Days	Vel. Km/Sec.	Intensities					
					Hδ	4202	4308	*Hγ*	4571	*Hβ*
R Hydrae										
C 450	1920 May 31	9.0	+241	−17.2	.....	.....	0.4	.....	1	.....
532**	July 8	7.9	−124	−23.7	0.9	.....	.....	.....	.....	.....
γ 9890	1921 Jan. 30	6.7	+ 82	−27.0	0.3	.....	.....	0.5	.....	.....
9891	Jan. 30	6.7	+ 82	−18.2	4	1	0.6	5	0.6	Tr?
C 873	Jan. 30	6.7	+ 82	−25.0	3	0.8	0.4	5	0.6	.....
911	Feb. 25	7.7	+108	−24.4	2	4	5	8	7	3
964	Mar. 27	8.6	+138	−18.8	.....	2	4	1	6	0.6
1010	Apr. 29	9.3	+171	−20.4	.....	0.6	4	.....	7	Tr
1037	May 26	9.6	+198	−21.2	.....	Tr	1	.....	2	.....
1054**	June 19	(9.9)	+222	− 8.2	.....	.....	0.9	.....	3	.....
1562	1922 Feb. 13	5.6	+ 51	−22.4	6	0.8	.2	5	.....	0.4
γ 11509	1923 Jan. 4	5.6	− 31	−19.3	8	.....	.....	4	.....	.....
11513	Jan. 5	5.5	− 30	−20.9	8	.....	.....	4	.....	.....
12517	1924 Feb. 24	5.0	− 23	−22.3	14	.....	.....	10	.....	.....
C 2726	Mar. 24	4.0	+ 6	−15.8	12	0.6	.....	10	.....	3
γ 12583	Apr. 19	4.6	+ 32	−28.1	12	2	.....	12	.....	3
12588	Apr. 20	4.7	+ 33	−23.9	2	.....	.....	2	.....	Tr
12589	Apr. 20	4.7	+ 33	−26.5	2	.....	.....	2	.....	Tr
12667	May 20	5.9	+ 63	−28.2	8	2	0.6	10	0.5	4
12693	June 10	6.2	+ 84	−31.3	7	3	3	10	2?	5
12696	June 11	6.3	+ 85	−30.2	4	1	2	8	3	4
14194	1926 May 17	6.3	− 45	−15.3	9	.....	.....	2	.....	.....
14215	May 26	5.8	− 36	−18.1	10	.....	.....	3	.....	.....
14216	May 27	5.8	− 35	−14.5	11	.....	.....	3	.....	.....
C 3808	May 30	5.7	− 32	−12.1	11	.....	.....	3	.....	.....
3812	June 18	4.8	− 13	−12.4	9	.....	.....	6	.....	.....
3817	June 19	4.8	− 12	−24.9	2	.....	.....	2	.....	.....
3822	June 20	4.8	− 11	−17.6	2	.....	.....	0.8	.....	.....
3823	June 20	4.8	− 11	−12.1	(30)	.....	.....	(18)	.....	.....
γ 14269	June 21	4.7	− 10	−23.2	12	.....	.....	7	.....	Tr
14274	June 22	4.7	− 9	−28.6	9	.....	.....	6	.....	Tr
14275	June 22	4.7	− 9	−28.6	5	.....	.....	3	.....	.....
14276	June 22	4.7	− 9	−26.5	4	.....	.....	3	.....	.....
14277	June 22	4.7	− 9	−30.0	7	.....	.....	5	.....	Tr
C 3902	July 24	4.7	+ 23	−16.7	8	0.6	.....	6	.....	0.6
3903	July 24	4.7	+ 23	−17.0	4	.....	.....	3	.....	.....
3904	July 24	4.7	+ 23	−19.5	3	.....	.....	2	.....	.....
γ 14392	July 27	4.8	+ 26	−29.4	4	.....	.....	3	.....	.....
C 4175*	1927 Jan. 17	8.8	+200	−11.4	.....	0.9	2	.....	3	.....
4193*	Feb. 18	8.9	+232	.....	.....	.....	.....	.....	0.5	.....
4204*	Mar. 17	8.9	+259	.....	.....	.....	.....	.....	?	.....
4228	Apr. 14	8.4	−124	.....	.....	.....	.....	.....	.....	.....
4268*	May 13	6.9	− 95	−17.4	4	.....	.....	.....	.....	.....
4274	May 15	6.8	− 93	−31.9	2	.....	.....	0.2	.....	.....
4278	May 21	6.7	− 87	−18.8	3	.....	.....	0.6	.....	.....
γ 15061	June 18	5.9	− 59	−20.4	6	.....	.....	2	.....	.....
15068	June 19	5.9	− 58	−24.4	6	.....	.....	2	.....	.....
C 4318	June 22	5.8	− 55	−22.1	5	.....	.....	2	.....	.....
4323	June 23	5.8	− 54	−14.9	8	.....	.....	3	.....	.....

TABLE II—*Continued*

	Plate	Date	Mag.	Phase Days	Vel. Km/Sec.	Intensities					
						Hδ	4202	4308	*Hγ*	4571	*Hβ*
					R Leonis—*Continued*						
	2569	1923 Nov. 28	9.0	+118	− 0.3	.....	(0.3)	0.8	.....	1	.....
	2594	Dec. 24	9.7	+144	+ 0.9	.....	1	3	.....	5	.....
	2652*	1924 Jan. 23	10.2	+174	+ 2	.....	1	3	Tr	6	.....
	2798*	May 21	6.9	− 36	+ 3.8	10	.....	.....	2	.....	.....
	2802	May 22	6.8	− 35	+ 2.1	3	.....	.....	0.8	.....	.....
γ	14763	1926 Dec. 14	6.1	− 8	+ 0.8	8	.....	.....	4	.....	.....
C	4192	1927 Feb. 18	6.6	+ 58	− 3.6	5	0.8	.....	2	.....	.....
	4226*	Apr. 14	8.2	+113	− 1.0	2	4	4	2	4	.....
	4267*	May 13	8.8	+142	− 8.2	.....	1	2	.....	3	.....
	4273*	May 15	8.8	+144	− 1.6	.....	2	3	.....	4	.....
	4593	Dec. 23	6.6	+ 50	− 8.0	10	3	0.9	7	0.6	.....
	4612	1928 Jan. 4	7.0	+ 62	− 6.0	8	3	2	8	1	.....
	4616*a*	Jan. 5	7.1	+ 63	−12.0	3	0.9	0.4	2	.....	.....
	4616*b*	Jan. 5	7.1	+ 63	−12.1	2	Tr	.....	0.9	.....	.....
	4678*	Feb. 10	8.3	+ 99	+ 0.7	6	5	6	4	5	.....
	4681	Feb. 11	8.3	+100	− 2.7	6	6	7	5	8	.....
	4707*	Mar. 16	8.9	+134	+ 9.5	1	4	6	1	10	0.7
	4712	Mar. 17	8.9	+135	+ 6.4	0.2	2	3	0.4	5	.....
	4745*	Apr. 12	9.7	+161	+11.3	.....	0.6	2	.....	3	.....
	4748*	Apr. 14	9.8	+163	− 7.5	.....	5	8	0.9	14	.....
	4793*	May 2	10.0	+181	+ 7.7	.....	0.2	1	.....	2	.....
	4796*	May 3	10.0	+182	+ 3.0	.....	0.9	2	.....	5	.....
	5088*	Nov. 29	7.4	+ 65	+ 3.9	7	2	1	4	1	.....
	5098	Dec. 20	8.6	+ 86	+ 0.4	2	1	1	1	2	Tr
	5129*	1929 Jan. 24	8.9	+121	+ 2.9	0.6	4	6	0.7	9	.....
	5141*	Feb. 28	10.2	+156	+11.6	.....	0.7	2	.....	3	.....
	5152*	Mar. 20	10.2	+176	+ 5.4	.....	(0.3)	0.6	.....	1	.....
	5172*	Apr. 26	9.7	− 92	.....	.....	.....	.....	.....	.....	.....
	5188*	May 22	8.6	− 66	.....	Tr	.....	.....	.....	.....	.....
	5191*	May 23	8.6	− 65	+ 7	0.6	.....	.....	.....	.....	.....
	5219	June 21	7.0	− 36	+ 5	1	.....	.....	0.5	.....	.....
					S Ursae Majoris						
C	369	1920 Apr. 8	8.5	− 37	− 4.2	.....	.....	.....	2	.....	6
	373	Apr. 10	8.5	− 35	− 7.2	.....	.....	.....	4	.....	9
γ	9252	June 3	8.2	+ 19	− 6.6	.....	.....	.....	2	.....	5
	9848	Dec. 29	8.4	+ 18	−10.1	0.6	.....	.....	3	.....	4
C	866	1921 Jan. 28	8.7	+ 48	− 7.1	.....	.....	Tr	2	.....	4
	1169**	July 23	8.1	− 3	−13.4	.8	.....	.....	4	.....	8
	1553	1922 Feb. 11	8.5	− 26	+ 4.1	.....	.....	.....	3	.....	7
	1626	Mar. 19	7.6	+ 10	− 3.7	.8	.....	.....	5	.....	7
γ	11744**	1923 May 1	8.5	− 42	+ 6.2	.....	.....	.....	4	.....	10
C	2286	June 4	8.0	− 8	− 0.6	0.8	.....	.....	4	.....	10
	2340	July 3	8.1	+ 21	− 6.8	2	.....	Tr	6	.....	10
	2648	1924 Jan. 22	8.0	− 19	− 5.8	0.8	.....	.....	6	.....	11
	2702*	Feb. 21	8.2	+ 11	− 1.1	0.7	.....	.....	5	.....	8
	3187*	1925 Mar. 3	8.5	− 55	+ 9.6	.....	.....	.....	4	.....	8

TABLE II—*Continued*

Plate	Date	Mag.	Phase Days	Vel. Km/Sec.	Intensities					
					Hδ	4202	4308	*Hγ*	4571	*Hβ*
R Hydrae										
C 450	1920 May 31	9.0	+241	−17.2	.....	.....	0.4	.....	1	.....
532**	July 8	7.9	−124	−23.7	0.9	.....	.....	.....	.....	.....
γ 9890	1921 Jan. 30	6.7	+ 82	−27.0	0.3	.....	.....	0.5	.....	.....
9891	Jan. 30	6.7	+ 82	−18.2	4	1	0.6	5	0.6	Tr?
C 873	Jan. 30	6.7	+ 82	−25.0	3	0.8	0.4	5	0.6	.....
911	Feb. 25	7.7	+108	−24.4	2	4	5	8	7	3
964	Mar. 27	8.6	+138	−18.8	.....	2	4	1	6	0.6
1010	Apr. 29	9.3	+171	−20.4	.....	0.6	4	.....	7	Tr
1037	May 26	9.6	+198	−21.2	.....	Tr	1	.....	2	.....
1054**	June 19	(9.9)	+222	− 8.2	.....	.....	0.9	.....	3	.....
1562	1922 Feb. 13	5.6	+ 51	−22.4	6	0.8	.2	5	.....	0.4
γ 11509	1923 Jan. 4	5.6	− 31	−19.3	8	.....	.....	4	.....	.....
11513	Jan. 5	5.5	− 30	−20.9	8	.....	.....	4	.....	.....
12517	1924 Feb. 24	5.0	− 23	−22.3	14	.....	.....	10	.....	.....
C 2726	Mar. 24	4.0	+ 6	−15.8	12	0.6	.....	10	.....	3
γ 12583	Apr. 19	4.6	+ 32	−28.1	12	2	.....	12	.....	3
12588	Apr. 20	4.7	+ 33	−23.9	2	.....	.....	2	.....	Tr
12589	Apr. 20	4.7	+ 33	−26.5	2	.....	.....	2	.....	Tr
12667	May 20	5.9	+ 63	−28.2	8	2	0.6	10	0.5	4
12693	June 10	6.2	+ 84	−31.3	7	3	3	10	2?	5
12696	June 11	6.3	+ 85	−30.2	4	1	2	8	3	4
14194	1926 May 17	6.3	− 45	−15.3	9	.....	.....	2	.....	.....
14215	May 26	5.8	− 36	−18.1	10	.....	.....	3	.....	.....
14216	May 27	5.8	− 35	−14.5	11	.....	.....	3	.....	.....
C 3808	May 30	5.7	− 32	−12.1	11	.....	.....	3	.....	.....
3812	June 18	4.8	− 13	−12.4	9	.....	.....	6	.....	.....
3817	June 19	4.8	− 12	−24.9	2	.....	.....	2	.....	.....
3822	June 20	4.8	− 11	−17.6	2	.....	.....	0.8	.....	.....
3823	June 20	4.8	− 11	−12.1	(30)	.....	.....	(18)	.....	.....
γ 14269	June 21	4.7	− 10	−23.2	12	.....	.....	7	.....	Tr
14274	June 22	4.7	− 9	−28.6	9	.....	.....	6	.....	Tr
14275	June 22	4.7	− 9	−28.6	5	.....	.....	3	.....	.....
14276	June 22	4.7	− 9	−26.5	4	.....	.....	3	.....	.....
14277	June 22	4.7	− 9	−30.0	7	.....	.....	5	.....	Tr
C 3902	July 24	4.7	+ 23	−16.7	8	0.6	.....	6	.....	0.6
3903	July 24	4.7	+ 23	−17.0	4	.....	.....	3	.....	.....
3904	July 24	4.7	+ 23	−19.5	3	.....	.....	2	.....	.....
γ 14392	July 27	4.8	+ 26	−29.4	4	.....	.....	3	.....	.....
C 4175*	1927 Jan. 17	8.8	+200	−11.4	.....	0.9	2	.....	3	.....
4193*	Feb. 18	8.9	+232	.....	.....	.....	.....	.....	0.5	.....
4204*	Mar. 17	8.9	+259	.....	.....	.....	.....	.....	?	.....
4228	Apr. 14	8.4	−124	.....	.....	.....	.....	.....	.....	.....
4268*	May 13	6.9	− 95	−17.4	4	.....	.....	.....	.....	.....
4274	May 15	6.8	− 93	−31.9	2	.....	.....	0.2	.....	.....
4278	May 21	6.7	− 87	−18.8	3	.....	.....	0.6	.....	.....
γ 15061	June 18	5.9	− 59	−20.4	6	.....	.....	2	.....	.....
15068	June 19	5.9	− 58	−24.4	6	.....	.....	2	.....	.....
C 4318	June 22	5.8	− 55	−22.1	5	.....	.....	2	.....	.....
4323	June 23	5.8	− 54	−14.9	8	.....	.....	3	.....	.....

TABLE II—*Continued*

Plate	Date	Mag.	Phase Days	Vel. Km/Sec.	Intensities					
					$H\delta$	4202	4308	$H\gamma$	4571	$H\beta$
R Hydrae—*Continued*										
γ 15074	1927 July 6	5.2	− 41	−21.0	4	.....	.....	3	.....	....
15079	July 7	5.1	− 40	−27.8	3	.....	.....	2	.....	....
C 4360	July 15	4.9	− 32	−20.1	8	.....	.....	5	.....	0.4
4363	July 16	4.9	− 31	−21.5	7	.....	.....	6	.....	Tr
4364	July 16	4.9	− 31	−26.6	4	.....	.....	3	.....	....
W Hydrae										
C 931	1921 Feb. 28	(7.3)	− 11	+25.8	8	2	.....	6	.....	....
970	Mar. 28	(6.5)	+ 17	+23.7	6	2	0.7	7	0.9	....
1033	May 25	(7)	+ 75	+24.4	0.4	0.8	1	1	2	....
1057	June 20	(8)	+101	+25.0	.....	1	2	0.6	4	0.7
1664	1922 Apr. 13	(7)	+ 8	+29.0	7	2	.....	5	.....	....
1701	May 15	(7.2)	+ 40	+27.3	4	2	1	6	1	.2
1744	June 11	(7.5)	+ 67	+39.3	.....	0.2	0.4	0.5	1	....
1751	June 15	(7.5)	+ 71	+27.9	0.3	.8	1	1	2	....
2073	1923 Jan. 8	(7)	− 87	+30.8	4	.....	.....	0.8	.....	....
2703*	1924 Feb. 21	(7.5)	(−64)	+37.3	10	0.8	.....	2	.....	....
2799*	May 21	(6.5)	(+26)	+30.0	4	2	1	8	1	0.7
2819*	June 13	(6.5)	(+49)	+29.2	2	3	2	4	2	2
2885*	July 18	(7.5)	(+84)	+32.9	.....	0.6	3	1	8	3
R Serpentis										
C 398	1920 May 1	7.6	+ 42	+ 3.1	6	0.4	.....	5	.....	0.3
447	May 30	8.8	+ 71	+ 6.1	6	2	1	6	0.7	.6
454	June 1	8.8	+ 73	+ 3.8	6	1	0.8	6	0.8	.6
533**	July 8	10.7	+110	+ 3.0	5	3	4	4	5	.6
1623	1922 Mar. 18	7.1	+ 15	+ 6.6	8	.....	.....	6	.....	.8
γ 11604	1923 Feb. 5	7.4	− 14	+10.1	4	.....	.....	2	.....	....
C 2145	Mar. 1	7.2	+ 10	+13.0	8	.....	.....	4	.....	0.4
2219	Apr. 30	8.4	+ 70	+ 3.6	4	.....	.....	2	.....	....
2727	1924 Mar. 24	7.3	+ 35	+ 8.2	10	0.6	.....	10	.....	2
R Cygni										
C 961	1921 Mar. 26	7.2	+ 31	−49.3	2	.....	.....	6	.....	14
965	Mar. 27	7.3	+ 32	−47.0	2	.....	Tr?	6	.....	15
1007	Apr. 28	8.9	+ 64	−44.8	2	Tr?	0.4	6	.....	14
1038	May 26	9.2	+ 92	−43.7	2	0.2	0.8	6	0.5	15
1055**	June 19	9.7	+116	−44.4	2	2	3	7	2	18
1171**	July 23	10.6	+150	−42.5	0.1	0.8	2	2	2	8
γ 11079	1922 May 17	6.8	+ 4	−49.0	0.8	.....	.....	4	.....	9
11117	June 14	8.0	+ 32	−51.1	2	.....	0.2	6	.....	15
C 1784	July 11	8.9	+ 59	−44.4	2	.....	0.3	6	.....	14
2245	1923 May 5	8.5	− 43	−39.4	Tr	.....	.....	0.9	.....	8

TABLE II—*Continued*

Plate	Date	Mag.	Phase Days	Vel. Km/Sec.	Intensities					
					Hδ	4202	4308	*Hγ*	4571	*Hβ*
R Cygni—*Continued*										
γ 11843	1923 June 5	6.4	− 12	−53.2	4	.....	.....	9	.....	19
11912	June 30	6.5	+ 13	−42.6	5	.....	0.8	9	.....	18
C 2411*	Aug. 30	8.5	+ 74	−48.0	4	1	2	11	.....	20
2474*	Oct. 2	9.8	+107	−37.1	0.9	1	2	5	2	11
2510*	Oct. 26	10.3	+131	−41.0	.....	1	3	2	3	7
2886*	1924 July 18	9.5	− 45	−39.4	0.2	.....	.....	3	.....	7
2918*	Aug. 9	7.8	− 23	−43.2	0.4	.....	.....	3	.....	9
3019	Sept. 16	8.2	+ 15	−43.9	2	.....	0.4	5	.....	14
3536*	1925 Sept. 30	7.4	− 33	−40.8	2	.....	.....	4	.....	12
3572*	Oct. 30	6.8	− 3	−45.4	2	.....	.....	6	.....	15
χ Cygni										
C 449	1920 May 30	8.2	− 38	−15.3	8	.....	.....	1	.....	0.6
455	June 1	8.1	− 36	−17.9	6	.....	.....	1	.....	Tr
γ 9315	June 30	5.2	− 7	−20.7	14	.....	.....	8	.....	7
C 559	July 29	5.8	+ 22	−20.6	15	2	0.6	8	.....	7
641	Sept. 3	6.5	+ 58	−18.8	12	2	2	8	0.9	4
669	Sept. 26	8.1	+ 81	−18.8	7	2	3	8	2	3
724	Oct. 26	9.6	+111	−13.0	2	4	5	4	4	2
1204**	1921 Aug. 13	5.4	− 5	.....	10	.....	.....	2	.....	.....
2362*	1923 July 27	9.4	−102	−18.8	0.6	.....	.....	.....	.....	.....
2407*	Aug. 29	7.8	− 69	−10.5	9	.....	.....	0.4	.....	.....
2438	Sept. 6	7.6	− 61	− 8.5	9	.....	.....	1	.....	.....
2466	Sept. 30	6.0	− 37	− 6.6	10	.....	.....	4	.....	2
γ 12232	Oct. 25	4.7	− 12	−17.8	16	.....	.....	9	.....	(8)
C 2561*	Nov. 26	5.2	+ 20	−15.2	12	0.8	.....	8	.....	(5)
DVI 83	1926 July 30	11.9	+178	.....	.....	1	3	0.3	3	?
C 4272*	1927 May 14	7.6	+ 63	−12.8	5	0.8	1	3	0.8	2
γ 14997	May 17	7.7	+ 66	−20.3	7	2	2	5	1	3
15065	June 18	9.1	+ 98	−14.2	2	0.8	1	1	0.9	0.2
C 4319	June 22	9.2	+102	−20.0	6	4	5	5	4	2
4353*	July 13	10.0	+123	−12.5	2	6	7	3	8	2
4379*	Aug. 13	11.2	+154	−13.4	.....	2	4	.....	5	.....
4413*	Sept. 8	12.0	+180	−10.3	.....	2	4	.....	6	.....
4747	1928 Apr. 13	5.5	− 23	−10.1	8	.....	.....	3	.....	2
γ 15709	Apr. 15	5.4	− 21	−21.0	6	.....	.....	3	.....	3
15710	Apr. 15	5.4	− 21	−21.0	8	.....	.....	4	.....	4
C 4757	Apr. 15	5.4	− 21	− 9.9	6	.....	.....	3	.....	3
4758	Apr. 15	5.4	− 21	− 8.6	7	.....	.....	4	.....	4
4759	Apr. 15	5.4	− 21	−11.8	9	.....	.....	5	.....	4
4760	Apr. 15	5.4	− 21	−11.9	10	.....	.....	5	.....	4
4800*a*	May 3	5.0	− 3	.....	0.9	.....	.....	0.4	.....	Tr
4800*b*	May 3	5.0	− 3	.....	2	.....	.....	1	.....	0.8
4800*c*	May 3	5.0	− 3	.....	7	.....	.....	4	.....	3
4800*d*	May 3	5.0	− 3	.....	10	.....	.....	7	.....	6
4866	June 28	6.6	+ 53	.....	10	0.8	.....	6	.....	2
γ 15948	July 5	7.1	+ 60	−14.3	10	1	0.6	6	.....	3

TABLE II—*Continued*

Plate	Date	Mag.	Phase Days	Vel. Km/Sec.	Intensities					
					$H\delta$	4202	4308	$H\gamma$	4571	$H\beta$
χ Cygni—*Continued*										
15950	1928 July 6	7.2	+ 61	−19.6	8	2	1	6	.....	2
16051	Aug. 4	8.7	+ 90	−23.3	1	.....	.....	0.9	.....	.....
16052	Aug. 4	8.7	+ 90	−19.5	5	1	0.8	3	0.4	0.3
C 4958	Aug. 29	9.7	+115	−15.7	2	4	4	3	2	1
5021*	Oct. 4	11.2	+151	− 4.5	0.2	3	6	0.8	4	.....
5058*	Nov. 1	12.2	+179	+ 2.2	.....	1	2	.....	2	.....
R Cassiopeiae										
C 674	1920 Sept. 27	8.6	+ 55	+ 5.7	6	0.2	.....	3	.....	.....
735	Oct. 28	9.2	+ 86	+ 4.8	8	1	.....	3	(0.4)	.....
γ 11504	1923 Jan. 4	7.4	+ 32	+ 5.3	4	.....	.....	1	.....	.....
C 2069	Jan. 8	7.5	+ 36	+12.6	10	0.4	.....	4	.....	.....
2475*	Oct. 2	11.6	−143	.....	.....	.....	.....	.....	.....	.....
γ 14273	1926 June 21	6.1	− 1	+10.5	8	.....	.....	6	.....	.....
14280	June 22	6.1	0	+11.2	14	.....	.....	9	.....	.....
14281	June 22	6.1	0	+ 6.7	3	.....	.....	2	.....	.....
C 3844	June 22	6.1	0	+12.8	6	.....	.....	4	.....	.....
3912	July 25	7.4	+ 33	+11.8	10	1	.....	7	.....	0.4
3913	July 25	7.4	+ 33	+ 5.8	6	0.2	.....	4	.....	.....
γ 14396	July 27	7.4	+ 35	+ 0.9	8	.....	.....	4	.....	.....
C 3994	Aug. 24	8.1	+ 63	+ 6.9	2	.....	.....	1	.....	.....
γ 14465	Aug. 25	8.2	+ 64	+ 6.4	6	.2	.....	4	.....	.....
14505	Sept. 16	8.6	+ 86	+ 3.6	3	0.5	.....	2	.....	.....
C 4013	Sept. 17	8.6	+ 87	+ 3.5	6	2	0.8	5	0.8	.....
4066	Oct. 22	9.4	+122	+ 7.1	6	4	4	5	2	1
4124*	Dec. 11	10.6	+172	+11.0	0.6	2	4	1	5	0.5
4326	1927 June 23	10.1	− 68	+ 5.0	0.6	.....	.....	.....	.....	.....
4355	July 13	8.7	− 48	+12.7	2	.....	.....	1	.....	.....
4423	Sept. 10	7.0	+ 11	+10.6	10	.....	.....	6	.....	.....
γ 15273	Sept. 16	7.1	+ 17	+ 3.2	8	.....	.....	4	.....	.....
15278	Sept. 17	7.2	+ 18	+ 5.0	2	.....	.....	0.8	.....	.....
15279	Sept. 17	7.2	+ 18	+ 3.4	2	.....	.....	0.5	.....	.....
C 4537	Nov. 10	8.2	+ 72	+ 7.4	7	0.5	.....	3	.....	.....
4618*	1928 Jan. 6	9.7	+129	+18.6	9	3	2	4	1	.....
5027	Oct. 5	7.4	− 31	+11.6	5	.....	.....	2	.....	.....
5028	Oct. 5	7.4	− 31	+18.1	2	.....	.....	0.6	.....	.....
γ 16239	Oct. 6	7.3	− 30	+12.0	2	.....	.....	0.7	.....	.....
16240	Oct. 6	7.3	− 30	+15.1	7	.....	.....	2	.....	.....

suited to accurate measurement of position. Fractional intensities indicate badly underexposed images; intensities greater than 10, overexposed images. The values given in Table II are the means of several estimates but cannot lay claim to high accuracy.

It is not feasible to pass from the apparent intensities in Table II to the actual intensities in the light of the variables, for the reason

that length of exposure, seeing, transparency of the air, guiding, slit-width, and other factors cannot be taken into account with sufficient accuracy. The *relative* intensities of two lines in the same spectral region may, however, be determined within reasonable limits of uncertainty. In this paper, accordingly, attention is given to the intensity ratios of pairs of lines, especially to the variations of such ratios with phase in the light-cycle. The data are suitable for a general comparative survey rather than for an exhaustive investigation of individual stars.

The apparent intensity ratios as estimated from the spectrograms have been used without corrections. Hence it is doubtless true that the data contain systematic errors arising from instrumental and photographic effects, but these errors are probably not extremely large. Moreover, the interest lies not so much in the actual values of the ratios as in their variations with phase, and here systematic errors are not important.

The ratios used in the numerous plots which exhibit the data were not calculated from the figures in Table II, but were estimated directly from the spectrograms. While this was being done, however, the estimated intensities of individual lines were recorded, and the means of these values, rounded off to the nearest integer (except those less than unity), appear in Table II. In some of the figures, material not plotted has been used in determining the position of the ends of the curves. This consists of plates on which only one line of the pair is well seen, but from which a limiting value may be obtained.

For some of the pairs of lines the ratios vary from small fractional values to three or more. In these cases the variations are exhibited better by using logarithms of the ratios rather than the ratios themselves; and to make the plots uniform, logarithms have been used throughout.

THE INTENSITY RATIO $H\gamma:H\delta$

In Figures 1 and 2 the logarithms of the ratio $H\gamma:H\delta$ from individual plates are plotted against the phase counted in days from light-maximum.

o Ceti.—The data for the plot (Fig. 1) were taken largely from

Table IX of Joy's paper,[1] supplemented by additional unpublished estimates which he has been kind enough to make available to us. The spectrograms were taken during the years 1916–1926.

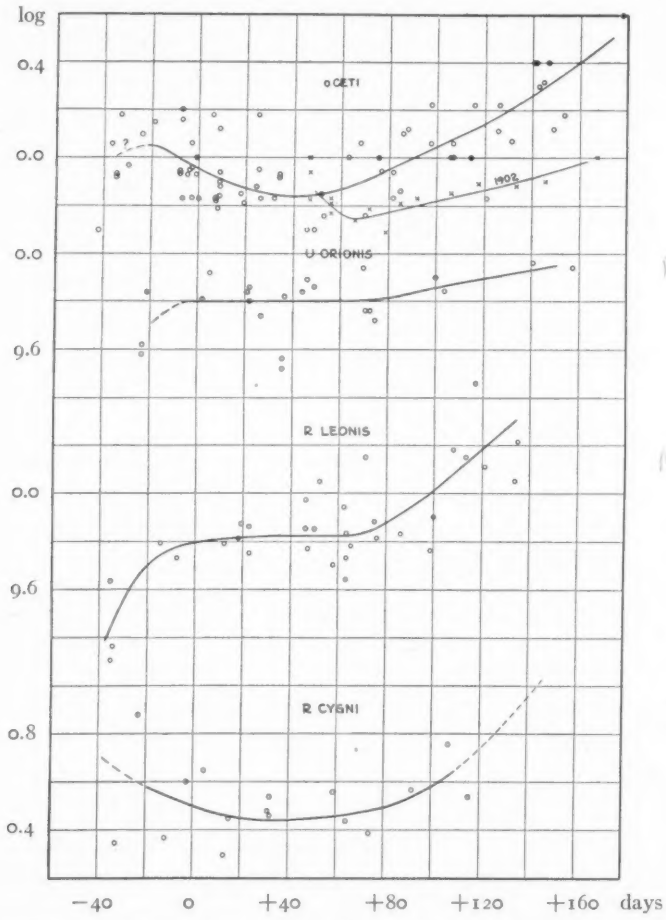


FIG. 1.—Intensity ratio $H\gamma : H\delta$. In the plot for o Ceti, circles represent estimates by Joy, crosses estimates by Stebbins.

The points near maximum and before it lie somewhat higher than for the other Me variables, with considerable scattering. The behavior is thus not quite typical. The beginning of the curve is probably raised by the influence of the bright lines of the companion,

[1] *Mt. Wilson Contr.*, No. 311; *Astrophysical Journal*, **63**, 281, 1926.

but this can scarcely be true at maximum, for at that time the bright $H\gamma$ and $H\delta$ lines belonging to the spectrum of the variable are overwhelmingly strong compared to these lines in the companion. At the right end of the plot the companion lines again play a part, but

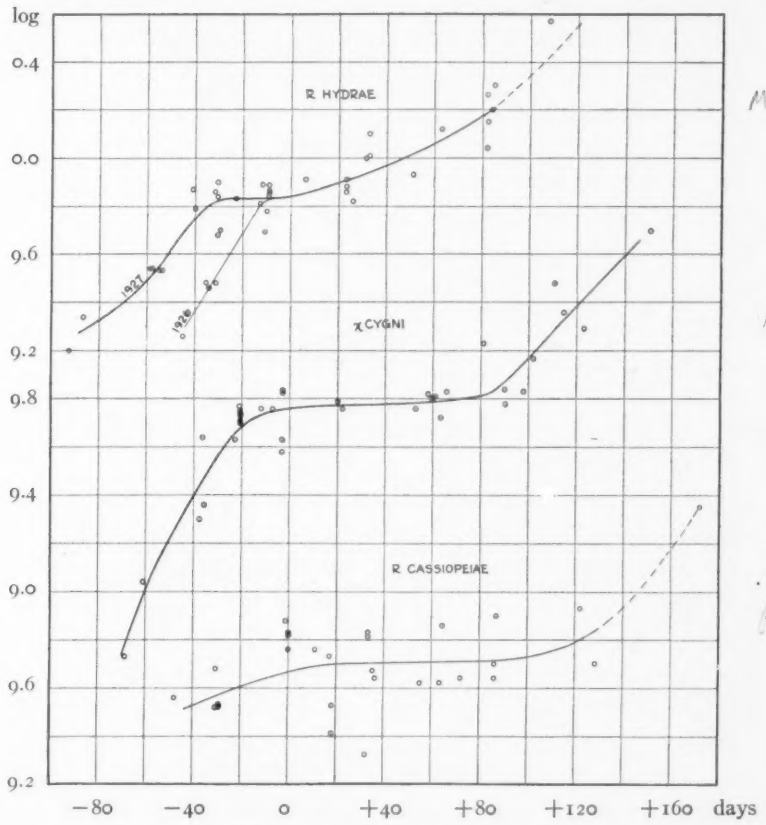


FIG. 2.—Intensity ratio $H\gamma : H\delta$

probably do not here greatly displace the curve, because at this phase the lines have nearly the same ratio in both sources.

Joel Stebbins' observations in 1902[1] are indicated by crosses. At phase +50 days they give the same ratio as Joy's data, but at later phases the ratios are decidedly lower than Joy's. This probably indicates that the scattering of points on the curves may be caused,

[1] *Lick Observatory Bulletins*, **2**, 78, 1903; *Astrophysical Journal*, **18**, 341, 1903.

not only by errors of observation, but by actual differences between individual cycles. Caution should of course be used in comparing intensities on spectrograms obtained with very different instrumental equipment. Joy's tabulated results[1] and those of W. J. S. Lockyer[2] show a slight dependence of the ratio on the magnitude at maximum, the ratio being higher at the brighter maxima. In a complete investigation of these variables it is obvious that the cycles must be studied separately.

U Orionis (Fig. 1).—The bright hydrogen lines are very intense. The number of observations at maximum and before is insufficient to determine the first part of the curve satisfactorily.

R Leonis (Fig. 1).—The form of the curve appears to be that characteristic of emission-line variables having spectra of a late subdivision of class M.

R Cygni (Fig. 1).—Several plates (omitted from the plot) on which $H\delta$ is very weak indicate that the curve rises at both ends, as shown by the dotted portions. For the distribution of the plates on the light-curve see Figure 1, *Contribution* No. 325.[3] The high value of the ratio $H\gamma:H\delta$ at phase -23 is derived from plate C 2918 taken before the low maximum of 1924. The only other plate at this maximum, C 3019, lies near the mean curve. The curve of this S-type star is decidedly different from those of the M-type variables. For a direct comparison see Figure 4.

R Hydrae (Fig. 2).—The observations of 1926 place the first branch of the curve considerably later, i.e., nearer maximum, than do those of 1925. Upon examining the light-curve we find that it had a narrower maximum in 1926 than in 1927, and that the slope immediately preceding maximum was steeper. $H\gamma$ appeared at about the sixth magnitude in both cycles, but the interval between the time of this occurrence and maximum was considerably less in 1926 than in 1927. This suggests that the sequence of the changes of the bright lines may be more closely related to certain features of the rising branch of the light-curve than to the time of maximum.

χ *Cygni* (Fig. 2).—This curve represents an unusually large

[1] *Op. cit.*, Table XIX.

[2] *Monthly Notices of the Royal Astronomical Society*, **84**, 563, 1924.

[3] *Astrophysical Journal*, **65**, 23, 1927.

change in the ratio, with a well-marked horizontal portion from maximum to phase +80.

R Cassiopeiae (Fig. 2).—The scattering of the observations between phases 0 and +40 may be due largely to errors of observation.

A number of stars in addition to those included in Figures 1 and 2 have reasonably well-determined curves for the ratio $H\gamma:H\delta$. These, with the individual observations omitted, are shown in Figure 3.

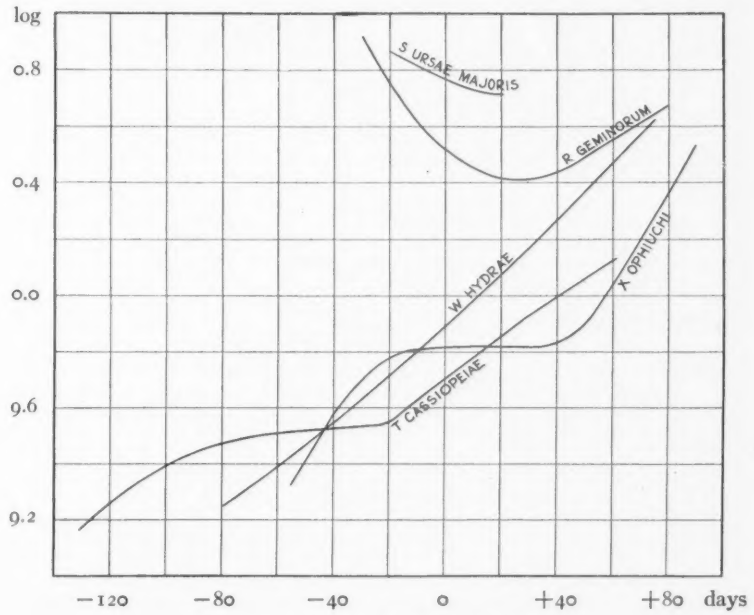


FIG. 3.—Intensity ratio $H\gamma:H\delta$

T Cassiopeiae (Fig. 3).—The flat portion of the curve is short, and the final rise begins at an earlier phase than in most of the Me stars. This may be connected with the unusually long interval from minimum to maximum. The bright hydrogen lines are less intense than in most advanced Me variables. The behavior of these and other bright lines seems to be slightly abnormal.

X Ophiuchi (Fig. 3).—This star is double,[1] but there is no evidence that the companion affects the light emitted by the variable star. The behavior of the ratio $H\gamma:H\delta$ seems to be normal.

[1] *Mt. Wilson Contr.*, No. 261; *Astrophysical Journal*, **57**, 251, 1923.

W Hydrae (Fig. 3).—The flat portion of the curve seems to be missing, the changes being hurried through quickly with no pause after maximum. This may have some connection with the unusually small light-range.

R Geminorum and *S Ursae Majoris* (Fig. 3).—The curves of these S-type stars resemble that of R Cygni (Figs. 1, 4).

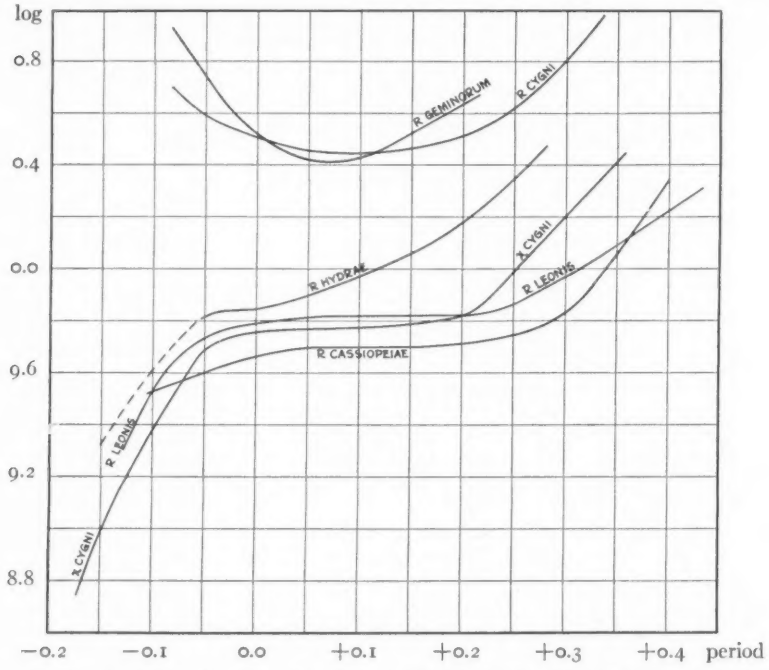


FIG. 4.—Intensity ratio $H\gamma:H\delta$, with phases in fractions of a period

To facilitate comparison, a few of the best-determined curves have been reduced to a common period and plotted together in Figure 4. The phases (abscissae) are here given in fractions of the period instead of days as in the other diagrams. The typical behavior of Me stars may be broadly outlined as follows. At one-fifth of the period before maximum, phase −0.2, the ratio is 1/8;[1] it rises rapidly to 1/2 at −0.05, then remains nearly constant until +0.2, after which it again rises rapidly to $2\frac{1}{2}$ at about +0.4. The behavior in Se stars is very different: The ratio has a minimum of $2\frac{1}{2}$ at phase

[1] The actual ratios are used here instead of logarithms.

+0.07, with considerably higher values both earlier and later. The value of the ratio obtained from a vacuum tube is slightly less than the highest found in Se spectra, but somewhat above the highest in class Me.

In Figure 5 the curves of the Me stars are replotted with the phases counted from minimum light instead of maximum. This procedure seems to bring them into somewhat closer agreement and

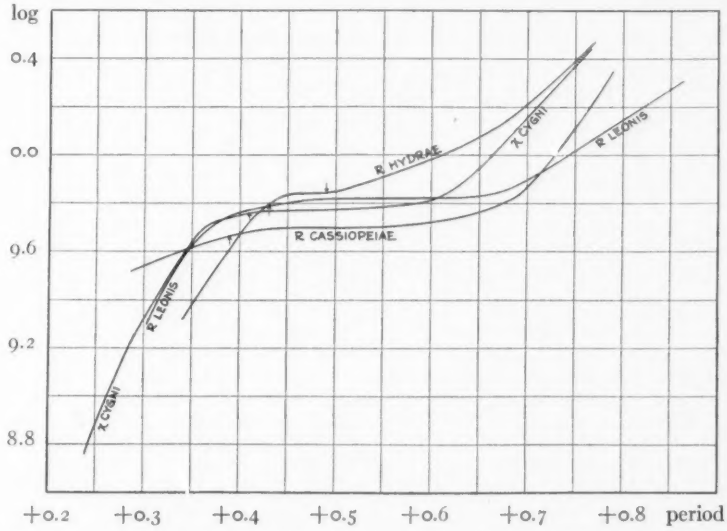


FIG. 5.—Intensity ratio $H\gamma:H\delta$, with phases in fractions of a period from minimum light. Arrows indicate times of maximum light.

suggests that in the periodic behavior of the bright lines, minimum is just as important as maximum; maximum may in fact be a mere incident and not a controlling event.

THE INTENSITY RATIO $H\beta:H\gamma$

In spectra of class Me, bright $H\beta$ is generally much less intense than $H\gamma$ or $H\delta$; in fact, in the later spectral subdivisions it is usually missing. Moreover, its intensity seems to be subject to large variations in different cycles. In general, therefore, the observations are scattering and do not yield well-defined curves for the ratio $H\beta:H\gamma$. In spectra of class Se, $H\beta$ is very intense. Figures 6, 7, and 8, and the following notes give some indication of the behavior of the ratio $H\beta:H\gamma$.

T Cassiopeiae.—One plate, C 5023, at phase +56 shows bright $H\beta$; another, γ 13163, at the same phase does not show it.

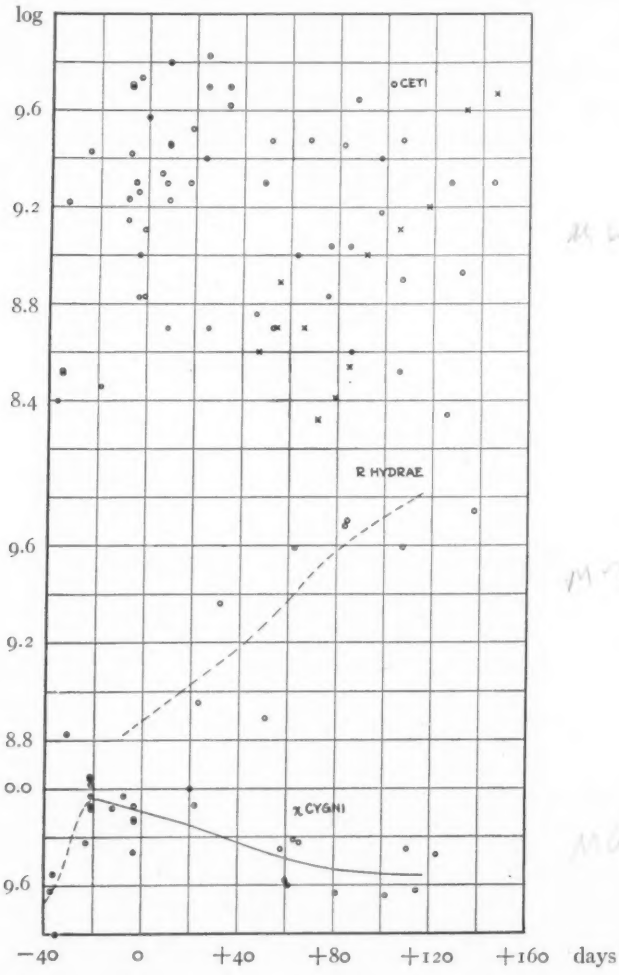


FIG. 6.—Intensity ratio $H\beta:H\gamma$. In the plot for *o* Ceti, circles represent estimates by Joy, crosses estimates by Stebbins.

o Ceti (Fig. 6).—The points are numerous but scatter irregularly over the diagram and do not form a well-defined curve. Joy's tabulation[1] shows a strong dependence on the maximum magnitude at-

[1] *Mt. Wilson Contr.*, No. 311, Table XIX; *Astrophysical Journal*, **63**, 281, 1926.

tained at each cycle, $H\beta$ being more intense at the brighter maxima. Stebbins' observations[1] in 1902 indicate a rapid increase in the ratio from phase +60 to +140 days. It is obvious that individual cycles must be studied separately.

R Hydrae (Fig. 6).—The meager material indicates a rapid and continuous increase after maximum.

W Hydrae.—A few observations indicate behavior similar to that of R Hydrae. The rise after maximum may be even more rapid.

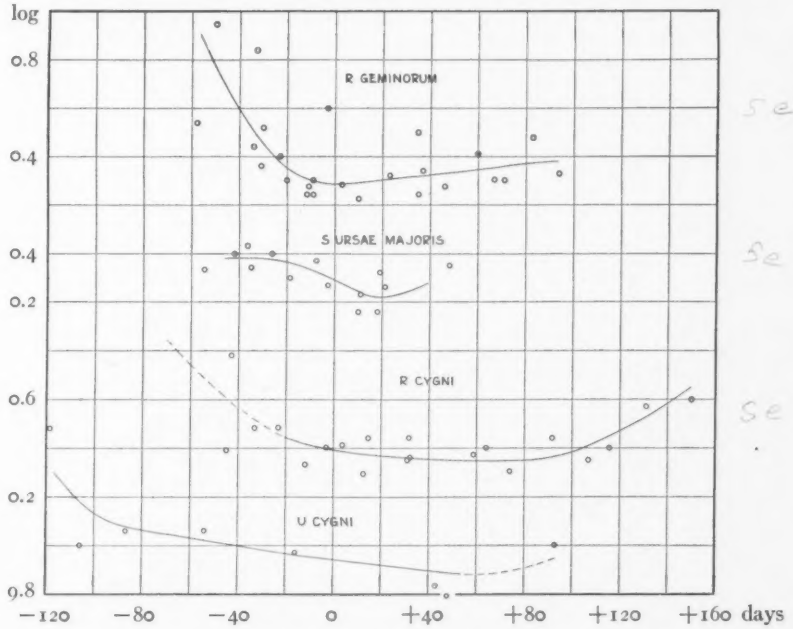


FIG. 7.—Intensity ratio $H\beta:H\gamma$

χ *Cygni* (Fig. 6).—The highest values are near phase −20, with a decrease instead of an increase after maximum. The behavior of the ratio thus more nearly resembles that in S-type spectra than that in class Me. $H\beta$ is, however, relatively weaker throughout in χ Cygni than in R Cygni and R Geminorum (see Fig. 8).

R Cassiopeiae.—$H\beta$ is well marked on only one plate, C 4066, phase +122 days. Traces are seen on two or three other plates taken after maximum.

[1] *Lick Observatory Bulletins*, **2**, 78, 1903; *Astrophysical Journal*, **18**, 341, 1903.

R Geminorum, S Ursae Majoris, R Cygni (Fig. 7).—The curves for these S-type stars are similar and distinctly different from those of class M (excepting χ Cygni). They are high at early and late phases, with a flat minimum after maximum light. The changes are not extremely large.

U Cygni (Fig. 7).—Class R8e, period 461 days: The curve, derived from C. D. Shane's observations,[1] is nearly flat for a consider-

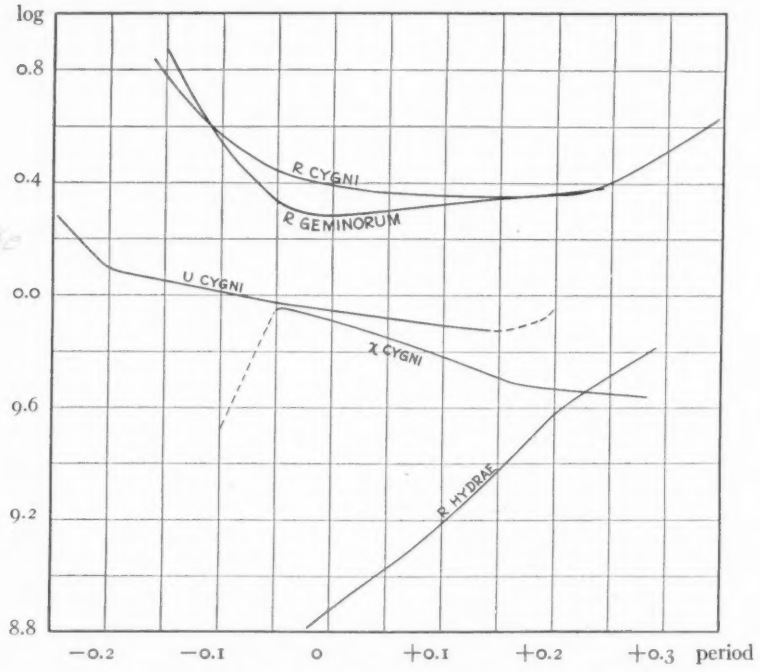


FIG. 8.—Intensity ratio $H\beta:H\gamma$, with phases in fractions of a period

able interval before and after maximum. The left end is higher, but at the other end the behavior is uncertain. In general shape, the curve resembles those of class S, but $H\beta$ is relatively less intense.

In Figure 8 several curves, plotted in fractions of the period, are brought together to facilitate comparison.

An experiment with a vacuum tube gave 0.4 for the logarithm of the ratio $H\beta:H\gamma$. This is comparable with most of the values for

[1] *Lick Observatory Bulletins*, **10**, 79, 1920.

S-type stars but higher than those for U Cygni (R8e) and the stars having titanium oxide bands.

McLaughlin and Petrie have recently made a statistical investigation of the bright hydrogen lines in long-period variables. They have used the intensity ratios $H\gamma:H\delta$ and $H\beta:H\gamma$, as we have done, and have studied their dependence upon spectral type and magnitude relative to minimum light. The brief abstract[1] of their paper at the Forty-first Meeting of the American Astronomical Society, which is the only account of their work available to us, does not indicate that they separated the observations before maximum from those after. Their results are therefore not readily comparable with those in the present *Contribution*, but we do not recognize serious discrepancies in the observational data of the two investigations.

THE INTENSITY RATIO λ 4308:λ 4202

We turn now to consideration of the bright lines at λ 4202 and λ 4308, generally believed to be iron lines. For a discussion of their identification see Joy's paper[2] on *o* Ceti. The periodic changes exhibited by these lines have been previously described.[3] The data concerning the ratio, derived in the same manner as for the hydrogen lines, are represented by Figures 9, 10, 11, and not much description is necessary.

In T Cassiopeiae the presence of λ 4202 long before maximum is an interesting feature probably connected with the large value of $M-m$.

In W Hydrae (Figs. 10 and 11), which has an abnormally small light-range for its period and spectral type, the bright lines λλ 4202, 4308, and 4571 develop more quickly after maximum light than in other Me variables.

The general behavior of the ratio in Me variables is as follows: λ 4308 is first seen at phase +0.1 or +0.2, and is then about one-fourth as intense as λ 4202. Its relative intensity increases at a steady rate, and it becomes equal to λ 4202 at about phase +0.3 and four times as intense at +0.55. The behavior in R Cygni, class

[1] *Popular Astronomy*, **37**, 270, 1929. [2] *Loc. cit.*

[3] *Ibid.*; *Mt. Wilson Contr.*, No. 200; *Astrophysical Journal*, **53**, 185, 1921; *Mt. Wilson Contr.*, No. 325; *Astrophysical Journal*, **65**, 23, 1927.

Se (Figs 9 and 10), is quite different. Whenever both lines are visible in this star, λ 4308 is about two and one-half times as intense as λ 4202. The ratio may have a minimum near phase +0.25. In the

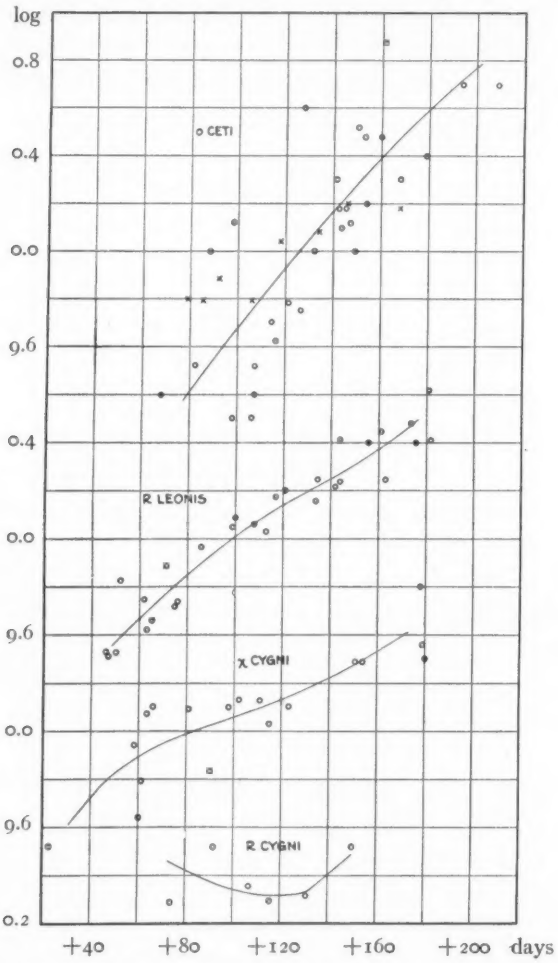


FIG. 9.—Intensity ratio λ 4308:λ 4202. In the plot for *o* Ceti, circles represent estimates by Joy, crosses estimates by Stebbins.

comparison spectrum (iron arc), λ 4308 appears twice as intense as λ 4202.

It happens in several stars that the lines $H\gamma$, λλ 4202, 4308, become of equal intensity at about the same phase. At this time $H\gamma$

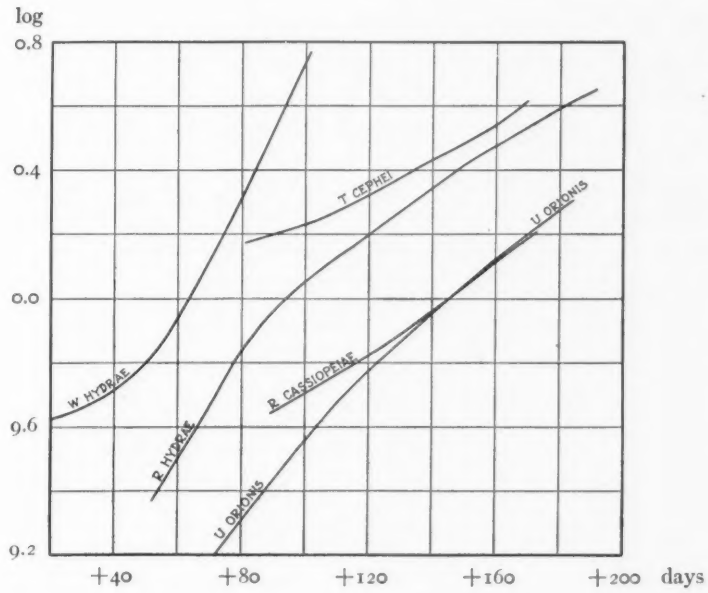


FIG. 10.—Intensity ratio λ 4308:λ 4202

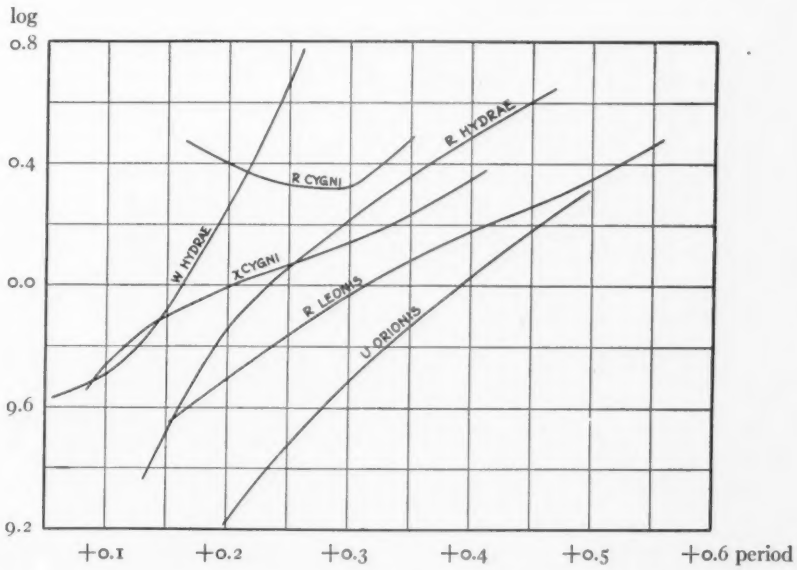


FIG. 11.—Intensity ratio λ 4308:λ 4202, with phases in fractions of a period

is decreasing in intensity with respect to the other two lines, while λ 4202 is decreasing relatively to λ 4308. The approximate mean phases in days and in fractions of the period at which the various lines become equal are shown in Table III.

THE INTENSITY RATIO λ 4571 : λ 4308

The low-temperature magnesium line λ 4571 is the last of the conspicuous bright lines to appear after maximum in each cycle. Toward minimum it frequently becomes the most intense line in the photographic spectrum. The behavior of the ratio λ 4571 : λ 4308 is

TABLE III

PHASES (AFTER MAXIMUM) AT WHICH VARIOUS LINES HAVE EQUAL INTENSITY

STAR	λ 4202 = $H\gamma$		λ 4308 = $H\gamma$		-λ 4308 = λ 4202	
	Days	Fraction of Period	Days	Fraction of Period	Days	Fraction of Period
U Orionis	155	0.42	161	0.43	145	0.39
R Leonis	91	.29	93	.30	98	.31
R Hydrae	122	.30	115	.28	94	.23
W Hydrae	87	.23	74	.19	64	.17
X Oph	(95)	(.28)	97	.29	95	.28
χ Cygni	117	.29	107	.26	87	.21
R Cass	142	.33	144	.33	145	0.34
R Cygni	(163)*	(0.38)	133	0.31	†	.....

* Extrapolated.

† λ 4308, whenever visible, is stronger than λ 4202.

displayed in Figures 12, 13, and 14. The changes are not so great as for the other pairs of lines, but λ 4571 tends steadily to become relatively stronger. Typical behavior may be outlined as follows: at phase +0.15, λ 4571 is two-thirds as intense as λ 4308; at +0.25 or 0.30 it is equal to λ 4308; while at +0.6 it is three times as intense.

T Cassiopeiae.—One plate, C 5023, at phase +56 days, shows λ 4571; another, γ 13163, at the same phase does not show it.

U Orionis (Fig. 13).—The intensity of λ 4571 at a given phase varies greatly in different cycles.

W Hydrae (Fig. 13).—The line λ 4571 is typical in behavior, except that it develops more quickly after maximum than in the other variables. This may be connected in some way with the unusually small light-range.

Comparisons of the behavior of a given ratio in different stars are possible by reference to Figures 1–14. In Figure 15 are plotted the curves of the various ratios in R Leonis, χ Cygni, and R Cygni.

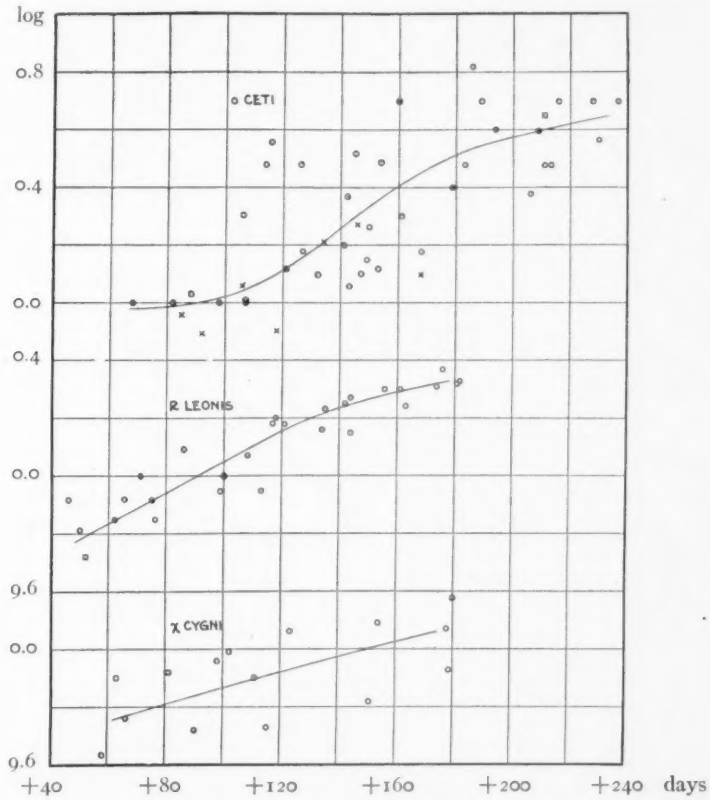


FIG. 12.—Intensity ratio λ 4571 : λ 4308. In the plot for *o* Ceti, circles represent estimates by Joy, crosses estimates by Stebbins.

WAVE-LENGTHS AND IDENTIFICATIONS OF LINES

In addition to the lines discussed in the preceding pages (namely, *Hβ*, *Hγ*, *Hδ*, λλ 4202, 4308, 4571), a considerable number of more or less prominent bright lines have been measured on many spectrograms. Most of these have already been observed in the spectra of *o* Ceti[1] and other long-period variables,[2] but the data in

[1] *Op. cit.*, Table VIII.

[2] *Mt. Wilson Contr.*, No. 265; *Astrophysical Journal*, **58**, 195, 1923; *Mt. Wilson Contr.*, No. 325; *Astrophysical Journal*, **65**, 23, 1927.

Tables V–VIII are more extensive than those previously published.

The mean wave-lengths in twelve stars are recorded in Table V. The wave-lengths measured on each plate were reduced for the

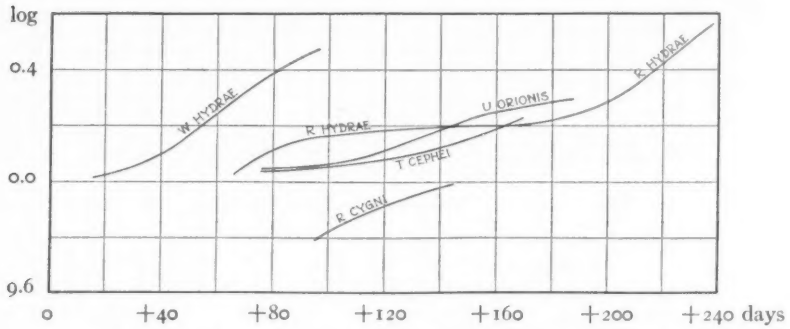


FIG. 13.—Intensity ratio λ 4571 : λ 4308

velocity determined for that plate from a selected list of emission lines given in Table IV. Each column of Table V includes the mean wave-length in international angstroms and the weight of the de-

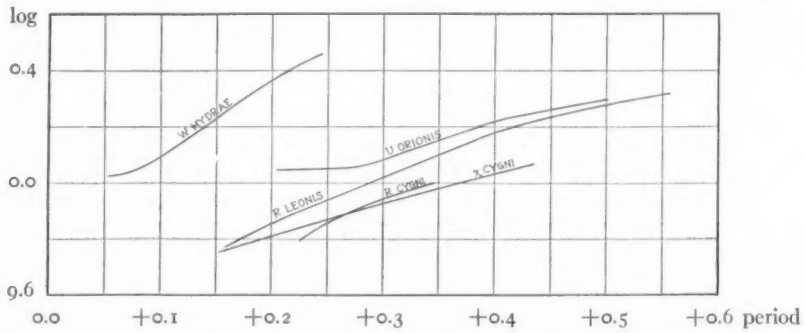


FIG. 14.—Intensity ratio λ 4571 : λ 4308, with phases in fractions of a period

termination. The weight rests on the following system: observations of standard quality, weights 1 and 0.5 for the 18-inch and 10-inch cameras, respectively; poor observations, weights 0.5 and 0.25. In printing, decimals .25 and .75 have been rounded to .2 and .8. To save space the measurements of T Cassiopeiae and the arguments for the whole table have been combined in the first column.

The wave-lengths given without decimals were not measured in T Cassiopeiae.

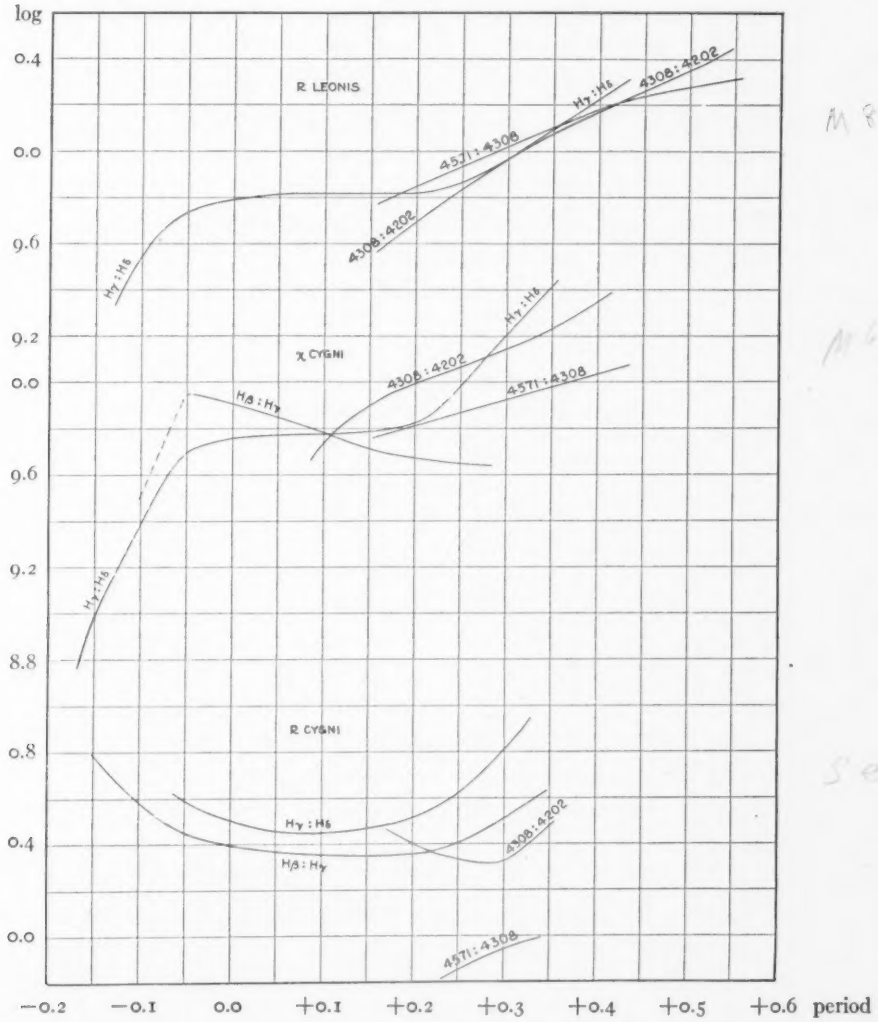


FIG. 15.—Intensity ratios of pairs of lines

T Cassiopeiae.—Lines especially strong before maximum are λλ 3905, 4103, 4511.

U Orionis.—The low-temperature iron line λ 4376 is especially strong on certain plates taken during the declining phase, but its

behavior varies greatly in different cycles. The variation of λ 4571 *Mg* in different cycles has already been noted.

The most interesting plate, perhaps, is C 2651, taken at phase +158 days, magnitude 10.3. The exposure, 4^h56^m, was sufficient to record numerous bright lines whose measured wave-lengths and probable identifications are in Table VI. The representation of iron lines is extraordinary. From the neutral atom, lines in temperature classes I, I A, and II appear, while the ionized atom contributes both normal and forbidden lines. The extreme low-temperature spectrum of neutral iron is well developed. Six of King's "persistent low-temperature lines"[1] lie in the well-observed region in U Orionis.

TABLE IV

EMISSION LINES USED FOR VELOCITY

I.A.	Ident.	I.A.	Ident.
3835.36	*Hη*	4202.03*	*Fe*
3889.05	*Hζ*	4307.91*	*Fe*
3905.51	*Si*	4340.47*	*Hγ*
3970.08	*Hε*	4571.11*	*Mg*
4101.74*	*Hδ*	4861.33	*Hβ*

* Used by Joy in his measurements of o Ceti.

Four of these were measured in the stellar spectrum, and there are suggestions of bright lines near the positions of the other two (λλ 4383, 4482). With the exception of λλ 4202, 4308, whose high intensity in these stars is an outstanding problem, λ 4376 is the strongest iron line, as was found by King to be true in the spectrum of the furnace at 1400°.

R Leonis.—Plate C 918, at phase +47, magnitude 6.9, exposure 132 minutes, emulsion Seed 23, is strongly exposed and shows numerous emission lines in the ultra-violet. The continuous spectrum is faint beyond λ 3900. The wave-lengths and intensities, with the probable identifications, are given in Table VII.

The lines of the magnesium triplet near λ 3835 are of low intensity, but their presence is of considerable interest. We are not aware that they have been previously observed in emission in stellar spectra, although it is well known that at a somewhat later phase the

[1] *Mt. Wilson Contr.*, No. 247; *Astrophysical Journal*, **56**, 318, 1922.

TABLE V

Collected Wave-Lengths of Bright Lines

T Cas	U Ori	R Leo	R Hya	W Hya	R Ser	χ Cyg	T Cep	R Cas	ο Cet	R Gem	R Cyg
3770	70.62 1	70.67 1	70.69 1			70.47 1			70.58 2		
3797	97.75 1	97.82 1	97.65 1			97.60 1			97.71 2		
3829		29.37 0.5									
3832		32.36 0.5									
3835	35.46 7	35.40 6	35.37 5		35.41 1	35.38 7	35.44 1	35.45 1.5	35.40 16		
3838	38.32 0.5	38.18 0.5									
3852	52.45 0.5	52.62 9				52.66 2		52.64 1.5	52.65 2		
3889.16 2.5	89.13 10	89.08 13	89.08 10		89.09 3	89.11 14.2	89.10 1	89.12 5	89.08 31	89.03 3.5	
3905.54 2	05.50 6	05.57 10.5	05.46 6.5		05.51 1	05.53 13.5	05.63 1	05.52 2.5	05.57 13		
3907		07.51 4							07.58 2		
3938	38.31 1	38.33 6.2				38.30 1		38.34 1.5	38.32 4		
3950	50.00 1	50.01 6.2				49.84 1.5		50.11 1			
3967		67.60 8.8				67.62 2		67.80 1	67.64 St.		
3970	70.07 2	70.08 9.5	70.02 1			70.16 2		69.97 1.5	70.07 11	70.07 1	
3977	77.63 1	77.77 11				77.86 2.5		77.73 1.5	77.9 St.		
4001		01.77 8.5	01.80 1			01.83 7		02.00 2.5	01.8 St.		
4006	06.77 1	06.92 1	06.83 5		06.85 1	06.76 7.5		06.80 2	06.74 9		
4030	30.34 0.5	30.49 10.5	30.53 1		30.70 0.5	30.51 7		30.55 1	30.57 2		
4101.73 11.2	01.73 14	01.72 23.8	01.74 25.5	01.73 7	01.74 6.5	01.70 25.5	01.70 3	01.74 14	01.74 51	01.69 15.5	01.69 13
4103.10 7.5	02.91 7	03.04 12	03.00 11.5	02.99 3	02.79 1	03.05 13		02.96 6	03.00 39		02.96 0.5
4138.75 1	38.51 6	38.69 3	38.66 15		38.50 3	38.64 16		38.62 8	38.60 23	38.53 3.5	38.68 2.5
4154		54.61 1	54.41 1		54.41 1	54.52 10.5				54.52 3	54.60 2
4165	65.87 1		65.82 9		65.76 2	65.78 3		65.96 3	65.84 16		
4178.93 2	78.87 10.5	78.86 13	78.76 14.5		78.67 3	78.79 19.5	78.75 1	78.78 8	78.82 35	79.04 3.5	79.02 3
4202.04 6.5	02.00 11	02.03 28	02.04 10.5	02.04 9.5	02.06 2.5	02.04 14	02.02 3.2	02.02 7	02.02 32	02.04 1	02.04 3.5
4206	06.59 0.5	06.51 3						06.70 1.8	06.70 10		
4215	16.00 1	15.95 8				15.60 4.8		16.08 0.5	16.19 18	15.57 1	15.27 1.5
4233	33.33 6.5	33.24 19.5	33.40 12	33.33 0.5	33.36 2	33.28 14.5	33.59 1	33.36 5	33.33 40	33.35 3.5	
4244	44.06 0.5	44.12 5				44.00 2					
4249.15 1	49.48 0.5	49.23 1.5	49.28 8	49.24 1.2	49.47 1	49.19 6		49.28 3.2	49.34 9	49.16 2.5	
4287	87.45 0.5	87.56 6.8	87.46 1.5			87.44 2		87.84 1.5			
4291	91.62 0.5	91.73 2				91.76 0.5		91.62 2	91.49 10		
4307.94 3.2	07.99 4.5	07.93 20.8	07.95 11	07.99 7.5	07.97 2.5	07.92 14	07.96 3.5	07.91 3	07.94 30	07.88 4.5	07.93 8.5
4340.46 10.5	40.44 14	40.44 26.5	40.45 26	40.43 10.5	40.43 6.5	40.46 25.2	40.47 4	40.45 14.5	40.48 58	40.51 19.5	40.50 15
4359		59.21 3				59.26 0.5					
4372.70 2	72.61 1.5	72.61 12.5	72.70 5	72.66 2	72.58 2	72.59 6.5	72.56 2	72.57 1.5	72.58 12		72.75 1.2
4375	75.88 5.5	75.85 19	75.90 3		75.87 2.5	76.08 2		76.03 4.5	75.93 31	75.94 0.5	75.99 1.5
4413		13.73 1				13.73 0.5					
4416		16.28 1				16.18 1					
4427	27.17 0.5	27.29 5						27.21 1	27.33 11		27.24 1
4461	61.61 0.5	61.50 1						61.62 1.5	61.59 9		61.71 0.5
4511.41 5.2	11.59 3.5	11.44 11	11.36 8	11.38 4.5		11.46 9.5		11.37 5	11.50 13	11.38 8.5	11.42 11.5
4521			21.36 5		21.41 2	21.60 5.5		21.31 2.5	21.40 4	21.44 15	21.48 12
4571.21 1	71.02 5	71.09 20.5	71.04 11	71.09 7	71.01 2.5	71.14 11	71.14 3.5	71.16 3.5	71.14 27	71.29 0.2	71.25 3
4861.34 1	61.29 2	61.40 4	61.34 12	61.40 3.5	61.28 4.5	61.32 21	61.27 1	61.30 1	61.31 26	61.31 15	61.33 12.5

magnesium line λ 4571 becomes very prominent in Me variables. The series relationships are as follows:

λ 3829.36	$^3P_0-{}^3D$	λ 3838.29	$^3P_2-{}^3D$
3832.31	$^3P_1-{}^3D$	4571.11	$^1S_0-{}^3P_1$

λ 4571 cannot be seen in the strong continuous spectrum on plate C 918, and it is weak or absent on C 910, taken the preceding night.

TABLE VI

WAVE-LENGTHS OF BRIGHT LINES IN U ORIONIS
(Plate C 2651; 10-Inch Camera)

STAR		LABORATORY		
I.A.	Int.	I.A.	Ident.	Temp. Class
4030.34*	0	.77	*Mn*	I
4101.60	2	.74	*Hδ*	.....
4178.91	0	.87	*Fe* II	.....
4202.08	3	.03	*Fe*	I
4206.59	1	.70	*Fe*	I A
4216.11	1	.18	*Fe*	I†
4233.27	0	.16	*Fe* II	.....
4244.06	0	3.97	[*Fe* II]‡	.....
4287.45	0.5	.40	[*Fe* II]‡	.....
4291.62	1	.47	*Fe*	I A
4308.11	4	7.91	*Fe*	II
4340.30	1.5	.47	*Hγ*	.....
4372.59	1	.....	?	.....
4375.93	1.5	.93	*Fe*	I†
4427.17	1	.31	*Fe*	I†
4461.61	0.5	.66	*Fe*	I†
4571.22	7	.11	*Mg*	I
4857.04	0	.....	?	.....
4861.31	0.5	.33	*Hβ*	.....

* Poor. Perhaps displaced by a dark line on the red side.
† Persistent low-temperature line.
‡ Forbidden line.

In general, however, this is just about the phase at which λ 4571 begins to be visible. It then strengthens rapidly. The fact that the ultra-violet triplet is not seen at later phases probably indicates that it does not increase in intensity. Unfortunately the observational data are meager, but so far as may be judged, the relative behavior of the ultra-violet lines and λ 4571 is what might be expected from the series relationships. Further observations of the ultra-violet triplet would be of value.

The forbidden lines of ionized iron, measured on one spectrogram

of U Orionis, are regularly seen in R Leonis toward minimum. They are just recognizable at phase +100 days, and thereafter gradually increase in intensity until at phases +160 to +180 they are well marked. They are thus strongest during the few weeks just preceding minimum. The emission of these lines requires the ionization of iron atoms, and their increase in intensity as the star grows cooler is not readily explicable. The paradoxical occurrence of these lines

TABLE VII

WAVE-LENGTHS OF ULTRA-VIOLET BRIGHT LINES IN R LEONIS

(Plate C 918)

STAR		LABORATORY	
I.A.	Int.	I.A.	Ident.
3770.67	2	.63	$H\iota$
3797.82	2.5	.90	$H\theta$
3829.37	0	.36	*Mg*
3832.36	0.5	.31	*Mg*
3835.33	4	.39	$H\eta$
3838.18	0.5	.29	*Mg*
3852.52	3	.....	.....
3889.03	5	.05	$H\zeta$
3905.55	3	.51	*Si*
3907.41	1.5	.....	.....
3938.33	2	.....	.....
3949.81	2	.....	.....
3967.34	1	.....	.....
3970.06	2	.08	$H\epsilon$
3977.73	2.5	.....	.....

in the same spectrum with the extreme low-temperature lines of neutral iron and λ 4571 of magnesium, as in U Orionis, has been repeatedly observed in R Leonis. The possibility that the ionized iron lines arise in a companion star, as in R Aquarii and o Ceti, suggests itself, but is not strongly supported by the observations. Perhaps the forbidden lines occurring just before minimum are in some way connected with the beginning of a new light-cycle.

R Hydrae.—The bright hydrogen lines are very intense in this spectrum. On plate C 3823, phase −11 days, the intensities of the ultra-violet bright lines are as follows:

λ 3905, 4; $H\zeta$, 9; $H\eta$, 5; $H\theta$, 3; $H\iota$, 3− .

Hι is almost as strong as *Hθ*, but the following line, *Hκ*, is not seen. Line λ 4511 is visible on a large number of plates from phase −95 to +108 days, but is weak or absent near maximum. Line λ 4521 is weaker, and was measured on only five plates. It is apparently not so much weakened at maximum as λ 4511. We may therefore judge that λ 4521 is a line of higher excitation than λ 4511. Line λ 4373 is especially conspicuous on plate C 911, phase +108 days.

W Hydrae.—Nearly all the phenomena of the bright lines seem to occur at earlier phases than in the other variables. The relatively small light-range may be responsible for this behavior.

χ Cygni.—This variable is of special interest, not only because of its large light-range, but also because the spectrum combines features of classes M and S. A low effective temperature is indicated by the intensity of the titanium bands and of the potassium doublet near λ 4045 and by other lines. The concurrence of the low effective temperature and the class S features leads to the belief that the density is exceptionally low, perhaps the lowest of any well-known star in the sky. Although χ Cygni is fairly bright at maximum (average magnitude, 5.1), the range is so great that it is difficult to obtain spectrograms near minimum (phase about +240 days). The plates taken nearest minimum are at phase +180 and are not strongly exposed. The chief features, however, are well shown. These include the bright lines ordinarily observed at the corresponding phases in other variables (namely, λλ 4202, 4308, 4571) and also the forbidden iron line λ 4244 with traces of other forbidden iron lines which accompany λ 4244 in η Carinae and other stars. On plate DVI 83, taken at phase +178 days, magnitude 11.9, the following bright lines were measured: 4202 *Fe*, 4244 [*Fe* II], (4250) doubtful, 4287 [*Fe* II], 4308 *Fe*, *Hγ*, 4359 [*Fe* II], (4371) doubtful, 4413 [*Fe* II], 4416 [*Fe* II], 4571 *Mg*, *Hβ*. In spite of the low dispersion, 185 A per millimeter at *Hγ*, most of the lines stand out clearly from the weak background of continuous spectrum.

o Ceti.—The data are largely from Joy's investigation. The lines used by him for velocity are those starred in Table IV and, in addition, the low-temperature iron lines λλ 4216.18, 4291.47, 4375.93. In the ultra-violet, Joy's values show a systematic deviation from laboratory measurements and from those by one of us in several

variable stars. Accordingly, the following corrections have been applied to Joy's wave-lengths in *o* Ceti:

λ 3770 to λ 3938	−0.12 A
3970	− .09
4006	− .06
4030	− .04
4101	0.00

The figures following Joy's wave-lengths in *o* Ceti in Table V give the number of plates on which each line was measured. A few lines not measured by Joy have wave-lengths from Stebbins' investigation[1] of *o* Ceti; these are marked "St."

The mean wave-lengths of the bright lines, together with their probable identifications, are listed in Table VIII. The values in parentheses refer to maxima in the continuous spectra which may or may not be emission lines. The others are thought to be true emission lines. The agreement between the stellar and laboratory values is reasonably good in most instances. The lines assumed to have their normal values in order to fix the radial velocity are listed in Table IV. Their actual measured values, reduced for the mean velocity, just as in the case of other lines, are used in Tables V, VI, VII, and VIII. Notes on several lines follow Table VIII.

DISPLACEMENTS OF THE BRIGHT LINES

Lines in the blue-violet region.—Previous investigations have shown that near maximum light the bright lines of long-period variables regularly yield apparent velocities algebraically less than those from the absorption lines. In a few stars, observations have indicated systematic changes in the velocities from the bright lines during the time of their appearance in each cycle.[2] The present investigation offers many new data which in general confirm the earlier results.

The measured velocities are found in the sixth column of Table II, and the wave-lengths upon which they depend are listed in

[1] *Lick Observatory Bulletins*, **2**, 78, 1903; *Astrophysical Journal*, **18**, 341, 1903.

[2] *Mt. Wilson Contr.*, No. 264; *Astrophysical Journal*, **58**, 215, 1923; *Mt. Wilson Contr.*, 311; *Astrophysical Journal*, **63**, 281, 1926; *Mt. Wilson Contr.*, No. 382; *Astrophysical Journal*, **69**, 379, 1929.

TABLE VIII

MEAN WAVE-LENGTHS AND IDENTIFICATIONS

Me	Se	Ident.	Me	Se	Ident.
3770.60	.....	.63 *Hι*	(4173.41)	.....	.48 *Fe* II?
3797.71	.....	.90 *Hθ*	4178.81	9.03	8.87 *Fe* II
3829.37	.....	.36 *Mg*	4202.03	.04	.03 *Fe*
3832.36	.....	.31 *Mg*	4206.66	.....	.70 *Fe*
3835.41	.....	.39 *Hη*	4215.....	.4	.52 *Sr* II?
3838.25	.....	.29 *Mg*	4216.11	.....	.19 *Fe*
3852.62	.....	.....	4233.3	.4	.16 *Fe* II
3889.09	.03	.05 *Hζ*	4244.08	.....	3.97 [*Fe* II]
3905.54	.....	.51 *Si*	4249.27	.16	.....
3907.53	.....	.....	4287.56	.....	.40 [*Fe* II]
3938.32	.....	.....	4291.58	.....	.47 *Fe*
3949.99	.....	.....	(4293.05)	(.47)	.....
3967.62	.....	.....	(4304.76)	.....	.....
3970.07	.07	.08 *Hε*	4307.94	.91	.91 *Fe*
3977.77	.....	.....	4340.45	.50	.47 *Hγ*
(3997.69)	.....	.....	4359.21	.....	.34 [*Fe* II]
4001.82	.....	.....	4372.61	.75	.....
4006.78	.....	.....	4375.91	.98	.93 *Fe*
(4010.4)	.....	.....	(4377.84)	(.4)	.....
4030.51	.....	.76 *Mn*	4413.73	.....	.79 [*Fe* II]
(4070.1)	.....	.....	4416.23	.....	.28 [*Fe* II]
4101.73	.69	.74 *Hδ*	4427.30	.24	.31 *Fe*
4103.01	.....	2.95 *Si*	(4458.76)	(.62)	.....
(4119.58)	.....	.....	4461.59	.71	.65 *Fe*
(4122.74)	.....	.....	4511.43	.40	.31 *In*?
4138.62	.59	.43 (*Fe* II calc.)?	4521.44	.46	.....
4154.52	.55	.....	4571.10	.25	.11 *Mg*
4165.83	.....	.....	(4578.83)	.....	.....
(4170.68)	.....	.....	4861.32	.32	.33 *Hβ*

NOTES TO TABLE VIII

3797 *Hθ*.—The wave-length of the stellar line seems to be 0.2 A less than the laboratory value. This discrepancy is probably greater than the error of measurement.

3852.—This is one of the most important unidentified lines. The phases at which it was measured are as follows: U Orionis, 1 plate, +37 days; R Leonis, 10 plates, +46 to +100; χ Cygni, 4 plates, +61 to +123; R Cassiopeiae, 2 plates, +122 to +129. On many plates it is stronger than *Hη* and on a few it is equal to *Hζ*.

3889 *Hζ*.—The wave-length in *Me* spectra is slightly greater than the laboratory value. This was previously indicated by the data in Table II, *Mt. Wilson Contr.*, No. 265; *Astrophysical Journal*, **58**, 195, 1923.

3907.—Although of low intensity, this looks like a real emission line.

4001.—This appears to be a real emission line.

4030 *Mn*.—This line first appears as a bright edge on the violet side of the manganese absorption line λ 4030. In R Leonis and χ Cygni its measured wave-length increases from 4030.4 at phase +60 days to 4030.7 at +160. The mean value given in Table VIII is probably smaller than the true wave-length of the line to which it corresponds, and the identification with *Mn* 4030.76 is probably correct in spite of the apparent discrepancy in wave-length.

4101 *Hδ*.—The wave-length in χ Cygni, R Geminorum, and R Cygni is slightly lower than the laboratory value.

4103 *Si*.—Thirteen plates of eight other Me stars give 4103.01. One 7-inch camera plate of R Cygni gives 4102.96.

4119.—Two plates of two other Me stars give 4119.40.

4122.—One plate of W Andromedae gives 4122.85.

4138.—Nine plates of five other Me stars give 4138.64.

4154.—In Me spectra this looks like a maximum in the continuous spectrum and has very seldom been measured. It is more conspicuous in the S-type stars and χ Cygni. It may not be a real emission line.

4165.—Three plates of three other Me stars give 4165.96. This may be a maximum in the continuous spectrum and not a real emission line.

4170.—One plate of R Cancri gives 4170.88.

4173.—U Orionis gives 4173.17 from two plates. Omitting this low value, the mean for the other stars in Table V is 4173.49. Two plates of two other Me stars give 4173.44. Although this line looks like a maximum in the continuous spectrum, it may nevertheless correspond to 4173.48 *Fe* II.

4178.—Nine plates of five other Me stars give 4178.76.

4215.—In χ Cygni and the two S-type stars, the wave-length is approximately that of *Sr* II 4215.52, and decidedly below the low-temperature iron line 4216.19. The value measured in χ Cygni was omitted in taking the mean for the Me stars. In the Me spectra the line generally occurs on the same plates as λ 4376 (clearly a low-temperature iron line) and is probably due largely to neutral iron.

4233.—This line is probably due in part to the iron spark line 4233.16, but the wave-length measured in the variables is apparently modified by other features. It lies in the wing of the broad absorption line *Ca* 4227. In U Orionis, R Leonis, R Hydrae, χ Cygni, R Cassiopeiae, and o Ceti the measured value is about 4233.43 near maximum and decreases to 4233.15± at later phases. In R Leonis the decrease continues until the wave-length becomes about 4233.0.

4287 [*Fe* II].—Omitting the high value given by R Cassiopeiae, the mean becomes 4287.49.

4340 *Hγ*.—Before maximum the wave-lengths in R Leonis, R Hydrae, W Hydrae, T Cephei, and R Cassiopeiae show a tendency to be a few hundredths higher than the laboratory value, but at later phases they are a few hundredths lower. This effect is not exhibited by T Cassiopeiae or χ Cygni.

4373.—This is an outstanding unidentified line measured at the phases indicated below:

Star	No. Plates	Phase Days
T Cas	2	+ 21 to + 56
U Ori	3	+100 to +158
R Leo	17	+ 46 to +182
R Hya	5	+ 63 to +138
W Hya	3	+ 40 to + 84
R Ser	2	+ 71 to + 73
χ Cyg	8	+ 58 to +180
T Cep	2	+ 77 to +109
R Cas	2	+122 to +172
X Oph	2	+ 87 to + 94

Two plates of X Ophiuchi give 4372.60.

4376 *Fe.*—Six plates of χ Cygni indicate a rapid decrease in wave-length from 4376.35 at phase +65 to 4375.75 at +150, but the observations are of low weight and the effect may be accidental. The mean value for χ Cygni 4376.08 would normally have weight 4, but this was reduced to 2 because of the scattering.

4377.—Most of the measures were made before maximum.

4413 and 4416 [*Fe* II].—These lines were measured as a blend on one plate of R Leonis and four plates of χ Cygni. The mean wave-length is about 4415.16, indicating that the line at λ 4416 is slightly stronger than the other.

4511 *In*?—This is one of the few emission lines to appear before maximum. It has been measured as follows:

Star	No. Plates	Phase Days
T Cas	6	−132 to − 39
U Ori	5	+ 70 to +141
R Leo	12	− 36 to +100
R Hya	9	− 87 to +108
W Hya	5	− 87 to + 17
χ Cyg	10	− 69 to +115
R Cas	6	− 31 to +129

4521.—This line has been observed as follows:

Star	No. Plates	Phase Days
R Hya	6	−45 to + 84
R Ser	2	+10 to + 42
χ Cyg	8	−37 to +102
R Cas	5	−31 to + 33

Two plates of two other stars give 4521.45. The observations indicate that this line requires higher excitation than the neighboring line λ 4511.

4578.—A maximum was measured here on numerous plates of Me variables. It may be part of the band structure.

Table IV. There are indications that the various lines do not at all times give precisely consistent velocities, but the differences are small and have been disregarded in the present study.

The velocities are plotted against phase in the light-cycle in Figures 16, 17, and 18. In most of the stars a definite variation, though of small amplitude, is shown. As in the intensity plots, the scattering of the points at a given phase is caused by errors of observation and by differences between individual cycles.

T Cassiopeiae (Fig. 16).—A straight line at velocity −24.5 km/sec. satisfies the observations sufficiently well. This indicates either that the velocity is constant or that it varies through a smaller range than in most of the other stars.

W Hydrae (Fig. 16).—The curve, which is but poorly defined by the meager data, seems to be of the typical form with a flat minimum at +32 days.

R Serpentis (Fig. 16).—The curve is dropping at phase 0, but the observations do not cover a sufficiently long interval to locate the minimum velocity. It probably occurs not earlier than phase +70 days.

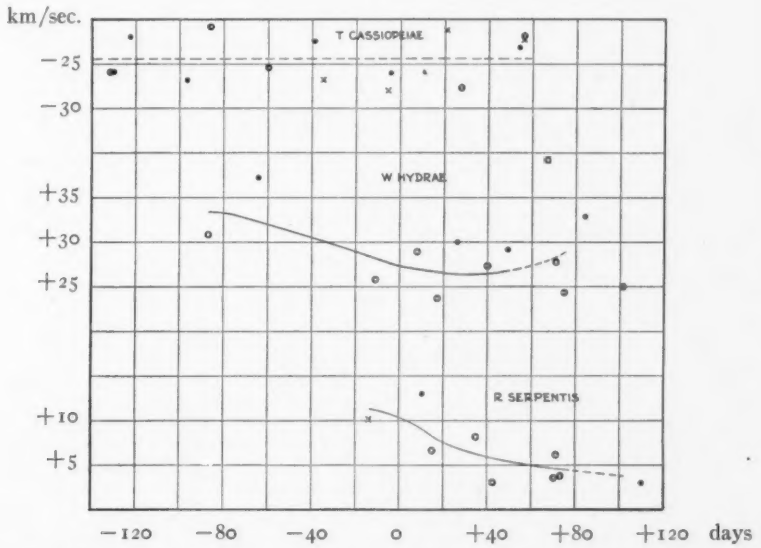


FIG. 16.—Velocities from bright lines. Circles represent observations made with the 100-inch telescope; crosses, observations made with the 60-inch telescope; small disks, observations of less than standard weight.

U Orionis (Fig. 17).—Normal places were formed as follows:

Phase Days	Velocity Km/Sec.	Weight
− 22	−31.5	1.5
+ 16	34.7	6
+ 41	37.1	6.5
+ 72	37.3	4
+102	33.9	2
+162	−28.5	1.5

Weights of 1 and 0.5 were given to observations of standard quality with the 18-inch and 10-inch cameras, respectively, while poor ob-

servations received one-half of these weights. Minimum velocity occurs near phase +55 days.

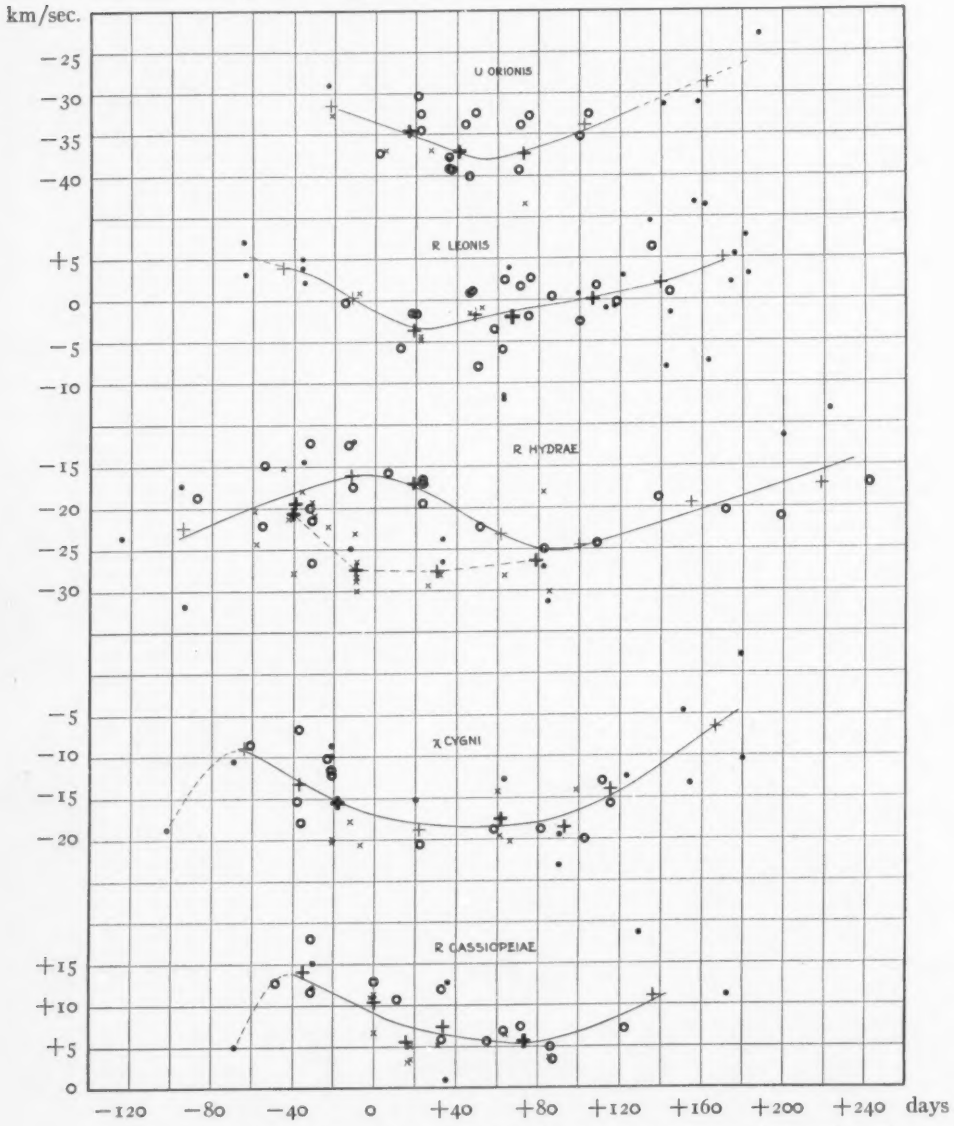


FIG. 17.—Velocities from bright lines. Circles represent observations made with the 100-inch telescope; small crosses, observations made with the 60-inch telescope; small disks, values of less than standard weight. Normal places are represented by large crosses, heavy symbols indicating high weight.

R Leonis (Fig. 17).—Normal places were formed as follows:

Phase Days	Velocity Km/Sec.	Weight
− 45	+3.9	2.25
− 12	+0.2	2
+ 19	−3.7	5
+ 48	−1.8	5
+ 67	−2.0	7.5
+106	0.0	6
+140	+2.0	3.5
+170	+5.0	3.25

Minimum velocity occurs near phase +25 days.

R Hydrae (Fig. 17).—The spectrograph attached to the 60-inch telescope seems to give algebraically lower velocities than that attached to the 100-inch. For this reason the two series have been treated separately. The cause of the systematic difference is not entirely clear. It may be partly an intensity equation and partly an effect of the southern declination. Other observers have found, in general, a negligible systematic difference between results obtained with the two telescopes.

Normal places formed from observations with the 100-inch spectrograph are as follows:

Phase Days	Velocity Km/Sec.	Weight
− 94	−22.5	2
− 39	19.6	6
− 12	16.2	3
+ 19	17.2	4
+ 61	23.3	1.5
+ 99	24.6	1.5
+154	19.6	2
+218	−17.3	2.5

Normal places formed from observations with the 60-inch spectrograph are as follows:

Phase Days	Velocity Km/Sec.	Weight
−40	−20.7	9.5
− 9	27.4	5
+30	27.6	3
+78	−26.4	4

The second normal place differs considerably from one in the other series at nearly the same phase. The five plates in the 60-inch normal place are apparently of good quality, but the four giving the lowest velocities were taken on one evening, and the fifth on the preceding evening. Hence it is barely possible that they are affected by some instrumental error which was especially active at that time.

The normal places from the 100-inch observations give a fairly well-defined curve with a maximum at phase −4 days and a minimum at +87 days. Both maximum and minimum velocity occur at earlier phases than in other stars.

χ Cygni (Fig. 17).—Normal places were formed as follows:

Phase Days	Velocity Km/Sec.	Weight
− 63	− 8.9	1.25
− 37	13.3	3
− 18	15.4	8
+ 21	18.8	1.5
+ 61	17.6	4.5
+ 93	18.6	4
+115	14.0	2.5
+166	− 6.5	2

The velocity-curve has a flat minimum near phase +49 days, and apparently has a maximum at −66 days.

R Cassiopeiae (Fig. 17).—Normal places were formed as follows:

Phase Days	Velocity Km/Sec.	Weight
− 35	+14.0	4
0	10.3	4
+ 16	5.6	4
+ 34	7.4	4
+ 73	5.5	7
+136	+11.0	2

The velocity-curve has a minimum near phase +67 days, and may have a maximum at −41 days.

R Geminorum (Fig. 18).—Three observations before maximum with the 60-inch spectrograph fall considerably lower than the 100-inch observations at the same phase. As these are the only 60-inch observations, it seems best, in view of a possible systematic error,

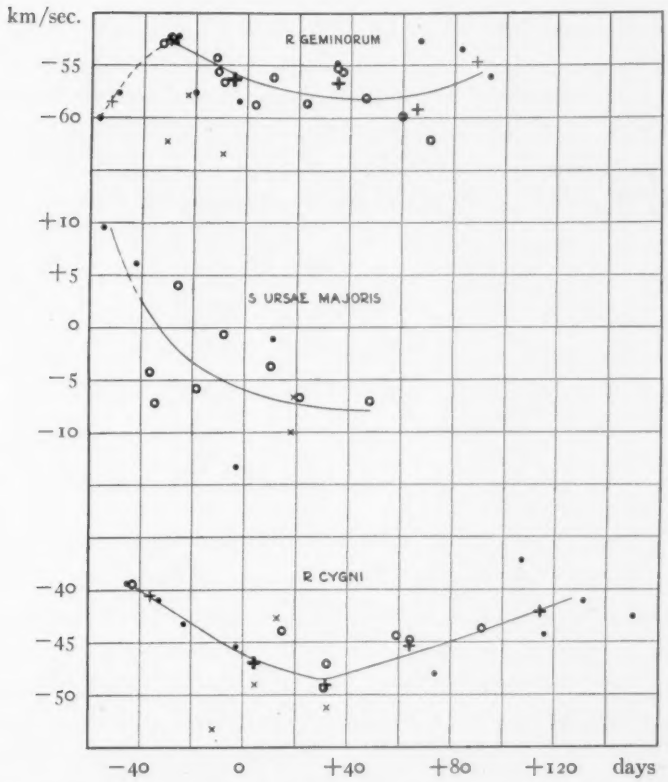


FIG. 18.—Velocities from bright lines. Circles represent observations made with the 100-inch telescope; small crosses, observations made with the 60-inch telescope; small disks, values of less than standard weight. Normal places are represented by large crosses, heavy symbols indicating high weight.

to omit them from consideration and base the curve on the 100-inch spectrograms. From them we find a curve resembling those for the Me stars, with maximum velocity at −30 days, and a flat minimum at about +46. Normal places were formed as follows:

Phase Days	Velocity Km/Sec.	Weight
−53	−58.4	.75
−30	52.6	3
− 5	56.4	5.75
+35	56.6	4.5
+66	59.2	2.5
+88	−54.8	1

S Ursae Majoris (Fig. 18).—The observations are rather scattering but appear to define a velocity-curve which falls more steeply before maximum light and through a larger range than do most of the other curves. This peculiarity may be connected with the relatively short period.

R Cygni (Fig. 18).—Normal places were formed as follows:

Phase Days	Velocity Km/Sec.	Weight
− 36	−40.6	2.25
+ 4	46.9	4.5
+ 32	49.1	3
+ 64	45.3	2.5
+115	−42.1	3

Minimum velocity occurs about 30 days after maximum light.

Two features of the displacement curve of the bright lines seem definitely established as typical of long-period variables, namely, the algebraically decreasing velocity at and shortly after light-maximum, and the flat minimum from 30 to 80 days after light-maximum. In several stars a maximum in the displacement-curve is indicated about 30 days before light-maximum, and this also is probably typical. The range in displacement is about 10 km/sec., probably being larger for the stars with the shorter periods. The phenomena seem to be essentially the same for Me and Se variables.

The bright Hα line.—Shane's observations of *o* Ceti during the maximum in 1919 indicate that "the hydrogen line *Hα* was displaced 1 A toward the violet with reference to the other hydrogen lines."[1] Joy found this result "to be confirmed, in part at least, by rough measures of the Mount Wilson plates."[2] It was not feasible in the present investigation to give this phenomenon the systematic observation which it deserves. A number of spectrograms were obtained, however, which considerably extend the previous data; these are listed in Table IX.

Most of the plates were taken with the plane-grating spectrograph (G series) having a dispersion of 67 A per millimeter; three

[1] *Lick Observatory Bulletins*, **10**, 134, 1922.

[2] *Mt. Wilson Contr.*, No. 311; *Astrophysical Journal*, **63**, 302, 1926.

TABLE IX

DISPLACEMENTS OF THE BRIGHT *Hα* LINE

Plate	Date	Mag.	Phase Days	Vel. Km/Sec.	Wt.	Remarks
		o Ceti				
G 47	1925 Jan. 3	3.8	+ 1	+35.0	0.5	
Coudé 37	1926 Oct. 15	3.2	− 4	+25.2	3	
Coudé 40	Oct. 16	3.2	− 3	+33.0	3	
G 198*b*	1928 Aug. 3	3.2	− 18	+30.1	1	
c	Aug. 3	3.2	− 18	+23.2	0.5	Film distorted?
G 207*b*	Sept. 1	3.0	+ 11	+43.8	0.5	Overexposed
c	Sept. 1	3.0	+ 11	+36.8	1	
G 209*a*	Sept. 1	3.0	+ 11	+29.6	1	
b	Sept. 1	3.0	+ 11	+30.5	1	
c	Sept. 1	3.0	+ 11	+26.4	1	
d	Sept. 1	3.0	+ 11	+33.2	1	
G 210	Nov. 30	6.6	+101	+33.6	0.5	Overexposed
G 215*a*	Dec. 1	6.7	+102	+29.3	1	
b	Dec. 1	6.7	+102	+32.2	1	
G 224	1929 Jan. 26	8.7	+158	+35.5	1	
Mean				+30.9	17	
		R Leonis				
G 221	1928 Dec. 1	7.6	+ 67	−16.0	1	
		χ Cygni				
G 186	1928 Aug. 1	8.5	+ 87	−37.8	0.5	Underexposed
G 193	Aug. 2	8.6	+ 88	−28.4	1	
G 246*a*	1929 June 20	5.1	+ 5	−38.4	1	
b	June 20	5.1	+ 5	−31.6	0.5	Underexposed
Mean				−33.8	3	
		R Geminorum				
G 41	1925 Jan. 2	6.3	+ 43	−63.6	1	
G 222	1929 Jan. 25	7.6	+ 37	−59.6	1	
Mean				−61.6	2	
		R Cygni				
γ 10078	1921 Mar. 29	7.4	+ 34	−53.6	1	
R 5	1923 June 2	6.5	− 15	−46.8	1	
R 12	July 1	6.6	+ 14	−38.8	1	
R 14	July 2	6.6	+ 15	−40.6	1	
G 25	1924 Oct. 4	8.5	+ 33	−57.2	1	
G 244	1929 June 20	7.9	+ 21	−50.7	1	
G 245*a*	June 20	7.9	+ 21	−43.4	1	
b	June 20	7.9	+ 21	−49.4	1	
Mean				−47.6	8	

plates of R Cygni, R5, R12, and R14, were taken with a three-prism spectrograph, dispersion at *Hα* 44 A per millimeter; two of *o* Ceti were with the coudé Littrow spectrograph having a 15-foot lens, dispersion at *Hα* 13 A per millimeter;[1] while one of R Cygni, γ 10078, was taken with a one-prism spectrograph and 40-inch camera, dispersion 70 A per millimeter.

TABLE X

MEAN DISPLACEMENT OF *Hα* RELATIVE TO OTHER BRIGHT LINES

Star	Displacement	
o Ceti	−15.9 km/sec.	= −0.35 A
R Leonis	14.8	= .32
χ Cygni	16.5	= .36
R Geminorum	− 3.6	= − .08
R Cygni	0.0	= 0.00

The measured displacements of the bright *Hα* line are given in Table IX. From each of these we have subtracted the value for the same phase read from the curves of the bright lines in the blue-violet region. The mean differences are given in Table X. The average displacement for the three Me stars is −0.34 A, while for the two Se stars it is practically 0.

On the following plates *Hα* is indistinguishable in the continuous spectrum:

Star	Plate	Date	Mag.	Phase Days
R Leonis Minoris	G 63	1925 Apr. 11	(7.0)	+ 9
R Leonis	G 60	Apr. 10	6.1	−17
R Hydrae	G 64	Apr. 11	5.7	−30
	G 69	June 8	5.1	+28
	G 71	June 9	5.2	+29

DISCUSSION

Dependence of spectroscopic features on the photosphere.—The intensities of absorption lines and bands in the spectra of long-period variables appear to change during the light-cycle in a way to be expected if the temperature of the photosphere and the reversing layer

[1] The coudé plates were taken by Messrs. Adams and Joy, to whom we are indebted for permission to use them.

rises and falls in phase with the brightness of the star. In Me variables, for example, it is well known that the titanium oxide bands and certain lines arising from the lowest atomic levels are less intense near maximum than at other phases. These lines include *Ca* 4227; *Cr* 4254, 4274, 4290; *K* 4044, 4047. In Se variables, *Sr* 4607 behaves in the same way. As far as present data go, the intensities of these features depend upon the magnitude of the variable rather than upon the phase. No systematic differences have been detected between the intensities on the ascending and on the descending branches of the light-curve.

It is important to notice that the foregoing remarks do not apply to the emission lines, for the intensities of these lines seem not to bear a simple relationship to the temperature of the photosphere. This somewhat surprising conclusion is supported by the following phenomena:

1. The intensity ratio $H\gamma:H\delta$, plotted as a function of the light-phase, has a total lack of symmetry with respect to the light-curve.

2. λλ 4202, 4308, 4571, and other lines that are conspicuous during the declining phase are not observable at the corresponding magnitudes *before* maximum.

3. Normal and forbidden lines of ionized iron are present near minimum when the absorption spectrum indicates a relatively low temperature for the reversing layer.

4. Absorption spectra practically identical with those of typical long-period variables, *but without bright lines*, are exhibited by numerous stars, many of which vary irregularly through small ranges. The strongest bright lines are found in the spectra of variables with long periods and large ranges.

5. The displacement-curve of the emission lines does not follow that of the absorption lines.

A few bright lines seem to be more dependent than the others on the brightness of the photosphere. These include λλ 4138, 4178, 4511, 4521.

The only recorded observation of emission bands in stellar spectra is that by Joy[1] of certain anomalous features in the spectrum of *o*

[1] *Mt. Wilson Contr.*, No. 311; *Astrophysical Journal*, **63**, 281, 1926.

Ceti in 1924, which were later identified by Baxandall[1] with aluminum oxide. Curiously enough, these emission bands appeared, not at an abnormally bright maximum, but at an abnormally faint one. This means that the magnitude at which the unusual bands appeared in 1924 was one through which the star regularly passes twice in each cycle. We must therefore assume the existence in 1924 of some influence other than the direct effect of the photosphere at a particular magnitude.

The effects of band absorption.—The important effects of the titanium bands on the spectral energy-curves of Me stars have been discussed by Joy and other writers. The absorption is undoubtedly more generally distributed along the spectrum than might appear at first glance. Even the stronger portions of the continuous spectrum within the region notched by the band heads are probably subject to band absorption. Thus a high-dispersion spectrogram of *o* Ceti clearly shows band lines in the strong continuous spectrum near λ 4615 at a point where the band absorption obviously is relatively weak.[2]

If light pressure on the atoms and molecules of a stellar atmosphere is proportional to the radiant energy absorbed, then titanium oxide must receive far greater upward impulses than any other substance known in M- and S-type stars, and thus may exist at an abnormally high level. The absorption-line spectrum, formed in the reversing layer immediately above the photosphere, would then be subject to modification by the overlying stratum of titanium oxide. This might consist merely of the superposition of the characteristic absorption bands; but if the stratum had sufficient optical thickness, it might be practically opaque in the spectral regions occupied by the bands, shutting off the original photospheric radiations in these regions and substituting for them radiations of the same wavelengths arising in the stratum itself. This would cause the disappearance of the low-level dark lines, or their reduction in intensity if the absorption in the high stratum were not complete.

That something of this sort actually occurs is indicated by the general absence or weakness of atomic lines within the region of the

[1] *Monthly Notices of the Royal Astronomical Society*, **88**, 679, 1928.

[2] See Plate Xe *Mt. Wilson Contr.*, No. 311; *Astrophysical Journal*, **63**, 281, 1926.

strong bands. This is brought out by a comparison of an advanced M-type spectrum, say M6, with one of type Mo or S. A considerable similarity exists except in the fluted region where the lines are wholly or partially suppressed in M6. The D lines of sodium furnish a striking example of the influence of the bands. These ultimate lines of the neutral sodium atom would be expected to strengthen with decreasing stellar temperature. This appears to hold true except for those stars (the coolest known) with very strong titanium bands; here the sodium lines are scarcely distinguishable with low dispersion, having but a small fraction of the intensity they exhibit in other red stars at somewhat higher temperatures. The ultimate strontium line λ 4607 shows the same phenomenon, although in a somewhat less marked degree, probably because the bands in its region are not so intense as those in the yellow. The same general effect may be observed in the spectrum of a long-period variable at different phases. As the bands increase in intensity after maximum the atomic lines become weak and hazy.

We may now inquire whether the bright lines of Me variables, as well as the continuous spectrum and dark lines, are subject to the absorption of a high stratum of titanium oxide. A more general question is the following: Are the peculiar relative intensities of the bright lines inherent, or are these lines emitted with normal intensities and then modified by absorption?

The hydrogen lines are the most important concerned in this problem. Not only are they very conspicuous, with relative intensities differing widely from those observed in the laboratory, but they undergo surprising systematic changes in each light-cycle, as described on previous pages. Several outstanding facts seem to point toward band absorption as the cause of the low intensities of $H\alpha$, $H\beta$, and $H\gamma$. The first is that no laboratory observation has shown these lines to be of as low intensity compared to $H\delta$ as they are in Me spectra, and no theory of atomic structure offers a plausible explanation of such an abnormality. On the other hand, radical modifications of the laboratory intensities are known only in stars with titanium bands.[1] Moreover, the degree of abnormality seems usually to be approximately proportional to the intensity of the

[1] *Mt. Wilson Contr.*, No. 306; *Astrophysical Journal*, 63, 13, 1926. See Table I.

bands. This is true, not only in a general comparison of one star with another, but also as applied to the various members of the hydrogen series in a given spectrum; that is to say, those lines are most affected which are in the region of the strongest bands.

Now if the hydrogen emission spectrum is subject to modification by the titanium oxide layer, it must be produced at a lower level. Is this physically possible? Ordinarily, emission lines are thought to originate in the outermost portions of stellar atmospheres. In the sun, for example, bright lines are produced by prominences which rise far above the level where absorption lines or bands are formed. Our knowledge of the structure of the atmospheres of giant stars is very incomplete, however, and we must not exclude from consideration far different circumstances in the Me stars. These objects are of enormous size and of extremely low density; their atmospheres, moreover, must be in a continual state of change and readjustment. Anomalous arrangements of various strata which would be impossible in other types of stars might well exist, at least for part of the light-cycle.

The assumption that the abnormally low relative intensities of the first three members of the hydrogen series are caused by the absorption of titanium oxide would seem to have sufficient merit to warrant its use as a tentative working hypothesis,[1] although it is by no means perfectly convincing and may eventually be rejected.

In applying this hypothesis to the data of the present paper we first note that $H\delta$ lies outside the region affected by the bands, that near $H\gamma$ the bands are of moderate intensity, and that near $H\beta$ they are very strong. The effect on $H\gamma$ should therefore always be less pronounced than that on $H\beta$, and this appears to be generally true.[2]

The curves for the ratio $H\gamma:H\delta$ begin with a low value (i.e., $H\gamma$ relatively weak), rise rapidly until about the time of maximum, after which they are nearly flat for several weeks, then turn upward

[1] This is not the first time that a hypothesis of this general nature has been introduced to account for irregularities in the Balmer series in Me stars. It has long been considered probable that the bright $H\epsilon$ line is absent because of the absorption of the calcium line H; and Shane has suggested that certain ultra-violet lines are weakened by absorbing vapors of iron and vanadium (*Lick Observatory Bulletins*, **10**, 134, 1922).

[2] The strong $H\beta$ line in the spectrum of χ Cygni may furnish an exception. This star is intermediate between classes M and S.

again and approach the normal (laboratory) value before the lines disappear. Whatever the mechanism, we may assume that $H\gamma$ is not at all times subject to the total band absorption but to a fraction of it, which becomes continually smaller as the phase advances, thus explaining a general rise of the curve from left to right. Superposed upon this effect, however, would be that of variations in the total band absorption. It is well known that in the spectra of Me variables the bands have their lowest intensities near maximum light. Thus before maximum the bands are decreasing in intensity and the two effects combine to give a rapid increase in the relative brightness of $H\gamma$. After maximum, as the bands are becoming more intense, the two effects act in opposite directions and thus give the curve a nearly horizontal portion.[1] At a still later phase the hydrogen lines largely escape the effect of the bands, and the curve rises again.[2]

It may be noted that on this hypothesis the observed time of transition from the first steep branch of the curve to the horizontal portion comes a little too early. If, through the whole period of visibility of the hydrogen lines, there were a tendency for them to approach their normal intensities (because of decreasing effective band absorption), we might expect the tangent to the curve at the time of maximum, when the band absorption is momentarily unchanging, to have a slope nearly corresponding to the average rate of change. As a matter of fact, however, the tangent at maximum is nearly horizontal; the tangent parallel to the general upward slope of the curve would occur a few days before maximum. We have then to assume that the reactionary effect of band absorption sets in, not exactly at maximum, but a few days before. It would be interesting if accurate measurements of band intensities should show that the bands begin to strengthen shortly before maximum.

The iron lines λλ 4202, 4308 belong to the same multiplet. Their classifications are

λ 4202	$a^3F_4-z^3G_4^\circ$
λ 4308	$a^3F_3-z^3G_4^\circ$

[1] In W Hydrae, whose light-range is unusually small (implying a small change in band intensity), the curve for $H\gamma:H\delta$ seems to lack the pronounced pause after maximum shown by most of the stars with larger ranges.

[2] A non-selective absorption increasing toward the violet may be a partial cause of the relative dimming of $H\delta$ at this phase.

The fact that the lines have a common upper level precludes certain explanations of their changing relative intensities. If the transition probabilities are constant under all circumstances—and this is the common belief—we are apparently forced to call upon changes in the absorption acting on the lines to explain the changing relative intensities shown in Figures 9, 10, and 11. The observed results might be caused by a decrease in the effective titanium band absorption after maximum.

Theory of long-period variables.—It is probable that much more observing and a more detailed organization of the data must precede the formation of a comprehensive theory of long-period variables. The present observations, while extensive, are not sufficiently precise or complete to answer many questions which naturally arise. One of the most evident needs is for a series of spectroscopic observations so frequent and complete that the spectral changes in a number of cycles may be thoroughly studied in their relationship to the details of the light-curve.

The behavior of the bright lines may, however, serve as a guide to certain general features that any finally successful theory must have. One suggestion is that, because of the large size of these stars or the low densities of their atmospheres, or for other reasons, various spectroscopic phenomena are not closely interrelated, but may proceed in partial independence of each other. The temperature of the photosphere, for example, appears not to have immediate control over the appearance and intensity of the bright lines. It seems rather as if the brightening of the photosphere were but one of a number of spectroscopic phenomena set going by a special occurrence at a certain phase. The temperature of the photosphere is probably closely correlated with the observed magnitude of the variable, but it does not dominate all features of the spectrum.

To illustrate the idea, let us postulate a release of energy in a certain level at intervals of time equal to the period of light-variation. These occur presumably just before minimum. The first visible result[1] is the brightening of the continuous spectrum, caused by either an increase in the temperature of the photosphere or a de-

[1] As an interesting speculation, we may ask whether the development of bright enhanced iron lines just before minimum is in some way a precursor of the new cycle.

crease in the absorption to which its light is subject. The aspect of the spectrum does not change materially for a time. A little later, however, there appears the first of a sequence of bright lines which runs throughout the remainder of the cycle while the photospheric spectrum is increasing to a maximum intensity and then fading. These two effects from a common cause apparently proceed with but little interaction. The intensities of the bright lines mark the phase unmistakably, but they seem not to bear a simple relationship to the temperature of the photosphere. The relative intensities of certain lines change in such a way as to suggest that the absorption to which they are subject becomes progressively less as the cycle advances. This may mean that the effective level of emission rises.

Among the lines to develop as the star decreases in brightness, one or two, particularly λ 4571 *Mg*, persist for a time *after minimum*, thus overlapping the new cycle. This indicates that the series of events started by some occurrence near minimum may last longer than a whole period. A similar phenomenon is furnished by the behavior of sun-spots. A given series of spots may remain visible for two or three years after the next cycle begins. This is another instance of periodic disturbances each having a chain of consequences longer than the period.

CARNEGIE INSTITUTION OF WASHINGTON
MOUNT WILSON OBSERVATORY
February 1930

A SPECTROGRAPHIC STUDY OF U SAGITTAE[1]

By ALFRED H. JOY

ABSTRACT

The spectrographic observations of U Sagittae, which extend from 1921 to 1928, were made with especial reference to the fainter component and the rotation effect.

The period.—A discussion of published photometric observations suggests a variable period with the elements: Min. = 2417130.4111 G.M.T. $+3.3806234E - 0.0057 \sin(0.^{\circ}2E - 34^{\circ})$. But for this study the elements used (Min. = 2424719.9007 G.M.T. $+3.38056E$) were derived from the spectrographic observations themselves.

The velocity-curve of the brighter star outside eclipse.—Forty-three spectrograms give the orbit of the principal star. Plates taken during eclipse were excluded. The spectrum, which has poor lines, is B9. The elements found are: $T = 1.5960$ days after minimum, $\omega = 260^{\circ}$, $e = 0.035$, $\gamma = -15.1$ km/sec., $K = 67.9$ km/sec., and $a \sin i = 3{,}155{,}000$ km.

Observations during totality.—The larger and fainter star alone is visible during totality, which lasts only one hundred minutes. Thirty spectrograms of the companion, taken with low dispersion, were used to determine the slope of the velocity-curve. The spectral type is G2. The mass is one-third that of the primary.

Observations during partial phases for rotation effect.—While the disk of the fainter star encroaches upon that of the brighter star, the velocities are affected by its rotation. With the inclinations and the duration of eclipse indicated by Fetlaar's uniform and darkened solutions, the values of the rotational velocities at the limb are 60 and 62 km/sec., respectively. The radius of the smaller star, corresponding to a velocity of 60 km/sec. at the limb, is 2,790,000 km.

Absolute dimensions and physical properties.—The principal physical dimensions were determined with the aid of Fetlaar's photometric solution. The uniform solution, which is more plausible, gives $a_1 = 3{,}169{,}000$ km; $a_2 = 10{,}457{,}000$ km; $a_1 + a_2 = 13{,}626{,}000$ km; $r_1 = 5.7\odot$; $r_2 = 3.4\odot$; $\rho_1 = 0.011\odot$; $\rho_2 = 0.171\odot$; $M_1 = +1.8$ mag.; $M_2 = -0.4$ mag.; $j_1 = +0.8$ mag.; $j_2 = -2.6$ mag.; $\pi = 0.''003$.

U Sagittae ($19^{h}14^{m}4$, $+19^{\circ}26'$, 1900; spectrum B9; 6.6–9.4 mag.) is one of the brighter eclipsing stars. It was discovered to be variable by Schwab in 1901, and a large number of photometric observations have been made by Wendell,[2] Luizet,[3] Graff,[4] Maggini,[5] and others. Russell, Fowler, and Borton[6] have given a photometric solution from photographic and visual observations made at the Harvard College Observatory. The extensive series of observations made by Nijland[7]

[1] *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 401.

[2] *Annals of the Harvard College Observatory*, **69**, 82, 1909.

[3] *Astronomische Nachrichten*, **162**, 285, 1903.

[4] *Mitteilungen der Hamburger Sternwarte*, **11**, 44, 1907.

[5] *Astronomische Nachrichten*, **200**, 54, 1915.

[6] *Astrophysical Journal*, **45**, 306, 1917.

[7] *Astronomische Nachrichten*, **176**, 167, 1907; **229**, 353, 1927.

has also been used by Fetlaar[1] for a determination of the elements of the photometric orbit. Hellerich[2] and Bemporad[3] have used their own observations, together with others, for special studies of the period. The spectroscopic orbit of the brighter star was determined by Miss Mary Fowler[4] from spectrograms made with the 30-inch reflector of the Allegheny Observatory.

Spectrographic observations of U Sagittae with the 60-inch and 100-inch reflectors were begun in 1921 for the purpose of studying the secondary star, which could be seen only when the star was faint at the time of total eclipse. It was hoped that such observations would reveal not only the spectral characteristics of the companion, but also that the duration of totality, which amounted to about one hundred minutes, would be sufficient for obtaining spectrograms which would give the slope of the velocity-curve with sufficient accuracy for a determination of the relative masses of the two components of the system. Later it was found necessary, for the sake of having consistent elements, to reobserve the whole velocity-curve. A study of the rotation effect affords a means of checking the absolute dimensions of the stars and gives opportunity for direct observational evidence on the question of darkening at the limb of an early-type star.

I. THE PERIOD

On account of the flat minimum of the light-curve during the time of total eclipse, variable-star observers have found it difficult to estimate the instant of mid-eclipse, yet there is considerable evidence that the period is not constant. Hellerich,[5] from an examination of the observations up to 1922, determined a mean period of 3.3806234 days, which seems to give the best linear representation of all the observations thus far published.

The observations used by Hellerich, together with additional minima by Luyten,[6] Gadomski,[7] and Mergentaler,[8] are listed in

[1] *Bulletin of the Astronomical Institutes of the Netherlands*, **3**, 195, 1926.
[2] *Astronomische Nachrichten*, **216**, 101, 1922; **221**, 29, 1924.
[3] *Contributi del R. Osservatorio Astronomico di Capodimonte*, No. 18, 1921.
[4] *Publications of the Allegheny Observatory*, **3**, 11, 1916. [5] *Loc. cit.*
[6] *Annals of the Leiden Observatory*, **13**, 55, 1922.
[7] *Circulaire de l'Observatoire de Cracovie*, No. 19, 1925.
[8] *Ibid.*, No. 25, 1927.

Table I. Weights have been assigned depending on the number of minima and the completeness with which both branches are cov-

TABLE I

OBSERVED MINIMA

Epoch	G.M.T.	O−C Unit, 0.0001 Day	Wt.	Observer
	J.D.			
− 426	2415690.2760	+105	1	Schwab
368	2415886.3407	− 10	1	Wendell
342	2415974.2438	+ 61	2	Luizet
335	2415997.9080	+ 57	3	Williams
− 333	2416004.6780	+145	1	Yendell
+ 74	2417380.5761	− 11	3	Graff
90	2417434.6649	− 24	4	Nijland
263	2418019.5168	+ 18	8	Nijland
497	2418810.5824	+ 44	3	Nijland
641	2419297.3861	− 46	1	Lehnert
746	2419652.3472	− 89	10	Maggini
757	2419689.5463	+ 33	1	Lacchini
845	2419987.0362	− 16	1	Lazzarino
859	2420034.3646	− 20	1	Lehnert
1198	2421180.4040	+ 61	3	Luyten
1566	2422424.4691	+ 18	3	Bemporad
1826	2423303.4417	+123	1	Hellerich
1831	2423320.3396	+ 71	1	Henz
1918	2423614.4452	− 16	5	Gadomski
+2365	2425125.5800	− 54	1	Mergentaler

ered by the observations. The residuals O−C in ten-thousandths of a day, computed from the elements,

$$\text{Min.} = 2417130.4111 \text{ G.M.T.} + 3.3806234E,$$

are given in the third column.

The observations of Table I may be combined into the normal places of Table II.

TABLE II

NORMALS

Epoch	O−C	Wt.
− 352	+67	8
+ 179	+ 1	15
753	−51	16
1382	+39	6
1892	+16	
+2365	−54	1

These normals are plotted in Figure 1, where the size of the circles varies inversely with the total weight of the point. The observations are closely represented by a sine-curve with a semi-amplitude of 0.0057 day and a period of 1800 epochs or 6085 days. The resulting elements are

$$\text{Min.} = 2417130.4111 \text{ G.M.T.} + 3.3806234E - 0.0057 \sin (0.^{\circ}2E - 34^{\circ}) .$$

While too much confidence must not be placed in this result, which is based on scattered observations with a wide spread, there can be

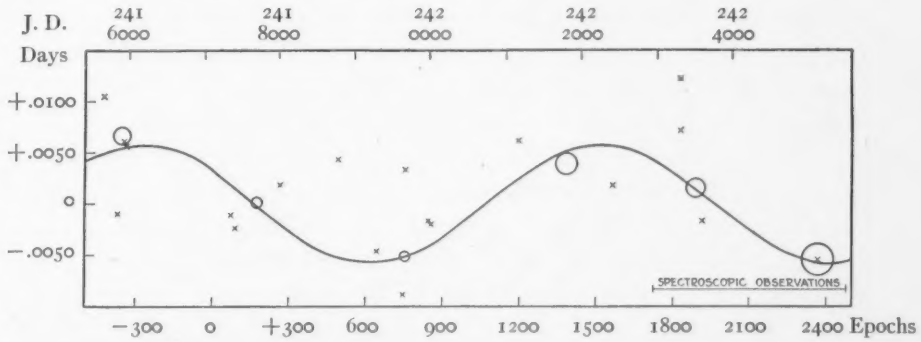


FIG. 1.—Observed minima of U Sagittae. The ordinates are residuals from the elements, Min. = 2417130.4111 + 3.3806234 E. The curve is computed from these elements with the addition of the term −0.0057 sin (0.°2 E − 34°). The crosses represent the observations of Table I. The circles, with diameters varying inversely with the weight, are the normals of Table II.

no doubt that there is a distinct variation in the time of minimum, which, in the mean at least, is periodic in nature. The accuracy of the observations does not justify the use of second harmonics.

The light-curves thus far determined show little indication of ellipsoidal figures for the stars, which would be expected if the change in period were due to revolution of the line of apsides of the system. Also, the components are rather widely separated for such a revolution in the comparatively short period of less than seventeen years.

If Figure 1 is a correct representation of the change in the time of minimum, the residuals from the linear elements should become increasingly positive from 1928 to 1936, corresponding to a lengthening

period. During the interval of the spectroscopic observations from 1921 to 1928 the period decreased in length.

Since neither the photometric observations nor the spectrographic results from 1921 to 1928 are completely represented by the elements suggested, even with the addition of the sine term, it seemed necessary in the present discussion to determine a period for this interval from the spectroscopic observations alone. The observations during totality are especially adapted to such a purpose. They fall on the steepest part of the velocity-curve, the range is considerable, the lines are numerous and suitable for accurate measurement, and the spectrograms had been taken in several series considerably separated in time. By trial a period was found which would make the sum of the squares of the residuals during totality the least. The results of such a method are much more reliable than those which would have been obtained by including the period in the least-squares solution of the principal star, whose spectrum is much poorer and has only one-third the velocity range of the secondary star.

The period obtained by this method has been used throughout the discussion of the spectrographic data. The elements used are

$$\text{Min.} = 2424719.9007 \text{ G.M.T.} + 3.38056E .$$

II. THE VELOCITY-CURVE OF THE BRIGHTER STAR OUTSIDE ECLIPSE

In order that observations covering the whole of the period might be made on a homogeneous system with the same instruments, spectrograms of the brighter component were taken outside of eclipse. On account of its faintness the spectrum of the secondary component does not appear at this time. These observations may be compared with those used by Miss Fowler[1] at the Allegheny Observatory. Such differences as occur are probably due to the errors of measurement of poor spectra with few lines rather than to any real change in the elements of the orbit.

In Table III are listed the forty-three spectrograms used in the solution of the velocity-curve of the principal star. The measured velocities are given, together with the weights used. Plates taken during partial and total phases are reserved for special treatment of the rotation effect and for the velocity-curve of the secondary star.

[1] *Loc. cit.*

The spectrum of the principal star is B9n. Helium lines are weak and only a few are visible. Measurement of the spectrum is difficult

TABLE III

OBSERVATIONS OUTSIDE ECLIPSE

Plate	H.G.M.T.	Phase	Vel.	Wt.	O−C
	J.D. 242+	days	km/sec.		km/sec.
C 4440	5159.6292	0.2557	−40	3	+ 4
γ 15700	5348.9583	0.2734	53	3	− 7
15143	5098.8201	0.2967	45	2	+ 3
C 3978	4750.6361	0.3104	37	2	+13
γ 15715	5365.9945	0.4068	53	2	+ 7
14354	4716.9993	0.4792	91	2	−25
16256	5548.6903	0.5524	66	3	+ 6
C 3982	4750.8791	0.5534	76	2	− 4
γ 15228	5129.6396	0.6911	75	2	+ 5
C 4737	5346.0333	0.7290	77	3	+ 4
γ 15230	5 9.7771	0.8286	92	3	− 9
14468	4754.6507	0.9444	81	2	+ 2
15134	5082.7576	1.1370	66	1	+10
15146	5099.6722	1.1488	61	3	+14
14356	4717.6695	1.1494	90	3	−14
14410	4727.8132	1.1514	74	3	+ 2
15135	5082.8514	1.2308	66	3	+ 2
14455	4751.6424	1.3167	73	3	−12
14519	4778.7181	1.3479	72	2	−14
C 4984	5515.7375	1.4052	53	1	− 2
γ 14360	4717.9910	1.4709	34	3	+10
C 3964	4748.6465	1.7013	−26	1	−13
4406	5130.7639	1.8154	+14	1	+12
4961	5492.6423	1.9739	16	3	− 5
γ 10430	2943.7055	1.9794	31	3	+10
15293	5164.7417	1.9876	28	3	+ 5
C 4387	5127.6785	2.1106	14	3	−22
γ 15158	5100.6750	2.1516	34	3	− 5
14361	4718.6813	2.1612	49	2	+10
14422	4728.9160	2.2542	57	3	+12
14459	4752.6479	2.3222	29	3	−19
15162	5100.9000	2.3766	57	3	+ 6
C 4029	4779.8229	2.4527	52	1	− 1
γ 14365	4718.9938	2.4737	63	2	+11
C 4436	5158.5979	2.6050	50	1	− 2
4742	5347.9792	2.6749	59	3	+10
3970	4749.6389	2.6947	61	2	+13
3972	4749.7007	2.7555	29	1	−16
3974	4749.8389	2.8937	36	2	+ 1
4966	5493.6319	2.9635	48	3	+17
γ 15301	5165.7347	2.9806	15	3	−14
15221	5128.6695	3.1016	8	3	−10
C 4941	5483.6368	3.1101	+20	2	+ 4

on account of the width and poor character of the lines, and the resulting velocities show large errors. The plates, usually of the

Eastman 40 emulsion, were taken with one prism and cameras of 18-inch and 10-inch focal length at the 60-inch and 100-inch reflectors. In assigning weights the general character of the spectrum was considered as well as the number of lines and the dispersion employed. The lines usually measured were H; K; $H\delta$; $H\gamma$; $H\beta$; λλ 4026, 4143, 4388, 4471, and 4481.

The total weight of the solution is considerably diminished by the exclusion of observations made during partial phases and the complete absence of the spectrum during totality.

The normal places of the velocity-curve are given in Table IV.

TABLE IV

NORMALS OUTSIDE ECLIPSE

Phase	Vel.	Wt.	O−C
days	km/sec.		km/sec.
0.3013	−45.6	12	+3.4
0.6076	−75.7	12	−0.9
1.0338	−89.6	12	+1.0
1.2330	−71.0	9	−2.5
1.5038	−39.0	8	+0.3
2.0129	+22.2	12	−3.4
2.1925	+46.2	8	+4.6
2.4102	+48.4	10	−3.1
2.7522	+50.1	8	+4.0
3.0325	+22.8	11	−0.4

The orbit is so nearly circular that values of T and ω could not be successfully determined by the usual method of least squares. Accordingly, a method of trial and error was employed for estimating the value of these quantities after the other elements had been determined by assuming a circular orbit. The resulting values are believed to be as accurate as can be expected in view of the nature of the observational material.

The final elements determined, as well as those of Miss Fowler from observations of 1907–1911, are given in Table V.

The differences between the two sets of elements is perhaps not larger than would be expected from such observations. The large change in the value of ω probably indicates that the eccentricity is too small to be determined from such poor data. The probable errors of a single observation of weight unity and an average normal place are 11.1 and 1.9 km/sec., respectively. The value of −15.1 km/sec.

found for γ has been determined entirely from observations of the principal star, but it has considerable additional weight, owing to the fact that the point of crossing of the observed primary and secondary curves has almost exactly this value.

III. THE OBSERVATIONS DURING TOTAL ECLIPSE

At primary minimum the larger star passes centrally or nearly so in front of the smaller. The duration of totality is about one hundred minutes. In this short interval of time it has been found possible, with the 100-inch reflector, to obtain spectrograms which give considerable knowledge of the spectrum and motion of the larger and fainter star. The dispersion employed is necessarily small, about

TABLE V

SPECTROGRAPHIC ELEMENTS OF U SAGITTAE

	Mt. Wilson 1926–1928	Allegheny 1907–1911
P (days)	3.38056	3.380603
T (days after min.)	1.5960	2.998
ω	260°	44°
e	0.035	0.035
γ (km/sec.)	−15.1	−19.13
K (km/sec.)	67.9	66.45
$a \sin i$ (km)	3,155,000	3,090,000

75 A per millimeter, and the exposure times are limited, averaging fifty-two minutes. Thirty spectrograms, as given in Table VI, have been taken which show measurable spectra of the fainter star. Some of these plates were exposed, wholly or in part, immediately before or after totality and give some indication of the presence of the continuous spectrum of the B-type star. Such plates are doubtless less reliable and have been given smaller weight.

The spectral type of the fainter star is gG2. The lines are remarkably good in view of the large rotation effect which must be present. The average number of lines measured is ten per plate.

Figure 2 is a detailed plot of the observations of the secondary star and portions of the two velocity-curves. The diameter of the circles varies inversely as the weight of the observations. There is some indication that the lines of the secondary are seen for a somewhat longer time before minimum than after, although the ap-

pearance of the spectra taken at equal intervals before and after minimum is closely the same.

TABLE VI

OBSERVATIONS OF FAINTER STAR

Plate	H.G.M.T.	Phase	Vel.	Wt.
	J.D. 242+	days	km/sec.	
C 1353	2968.6708	3.2808	−72.6	1
4287	5040.9701	3.2968	41.2	1
1354	2968.6916	3.3016	65.2	1
4827	5395.9354	3.3033	44.9	1
4033	4780.6764	3.3062	43.3	1
4900	5439.8930	3.3136	39.4	1
4288	5040.9938	3.3205	37.4	1
1355	2968.7174	3.3274	47.8	1
4265	5013.9590	3.3302	17.1	2
4299	5047.7715	3.3371	15.2	2
3891	4719.8632	3.3431	26.0	4
3858	4692.8201	3.3445	35.6	4
4034	4780.7181	3.3479	27.2	4
4828	5395.9847	3.3526	29.2	4
4901	5439.9403	3.3608	35.7	2
4300	5047.8000	3.3656	22.0	3
2903	3989.6847	3.3656	13.8	4
4266	5013.9952	3.3664	10.6	3
1273	2941.7132	3.3677	24.0	3
1356	2968.7583	3.3683	20.7	3
3892	4719.9007	0.0000	10.3	4
3859	4692.8618	0.0056	−14.9	3
4902	5439.9820	0.0219	+ 1.6	1
4301	5047.8438	0.0288	−12.2	4
4035	4780.7799	0.0291	·· 2.7	2
3893	4719.9375	0.0368	+12.5	1
2904	3989.7375	0.0378	− 6.6	2
3860	4692.8965	0.0403	− 0.7	3
4302	5047.8812	0.0662	+22.6	1
4991	5517.7826	0.0697	+ 0.9	1

Comparison of the slopes of the two velocity-curves gives the relative values of the velocity range and the ratio of the masses of the two stars. The mass ratio thus found is

$$\frac{m_1}{m_2}=3.3.$$

The value of K for the fainter star is $3.3\times 67.9=224.0$ km/sec. The individual masses in terms of the sun are 6.7 for the bright star and 2.0 for the faint.

IV. OBSERVATIONS DURING PARTIAL PHASES: ROTATION EFFECT

During partial phases of the eclipse before and after totality portions of the early-type star are covered by the G-type secondary. The eclipse is nearly central. As the eclipse proceeds, the light reaching the earth from the bright star has a more and more localized origin toward one limb until it is cut off entirely at the instant of totality. After the total phase has passed, the other limb is first exposed and then the whole disk is gradually uncovered. If the eclipse were exactly central, ideal conditions for observing the rotation effect would prevail, and the maximum effect could actually be observed if the observations could be continued until the instant of totality.

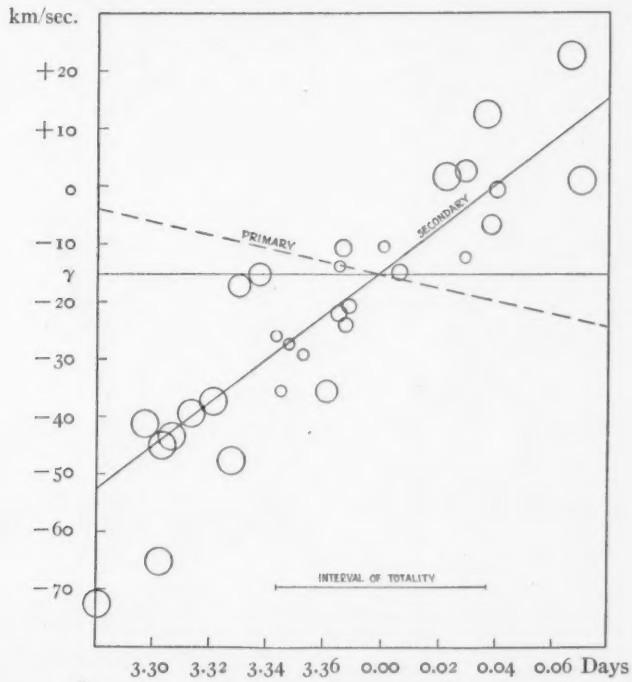


FIG. 2.—Detail of velocity-curves near totality. The size of the circles varies inversely with the weights. Comparison of the two slopes gives the relative masses.

Table VII is a list of the twenty-nine spectrographic observations during partial eclipse.

TABLE VII

OBSERVATIONS OF BRIGHTER STAR DURING PARTIAL ECLIPSE

Plate	H.G.M.T.	Phase	Vel.	Wt.
	J.D. 242+	days	km/sec.	
C 4898	5439.7181	3.1387	+ 8.3	3
4093	4834.6035	3.1443	23.5	1
γ 15222	5128.7181	3.1502	7.1	3
C 4094	4834.6229	3.1637	33.4	1
4825	5395.8208	3.1887	31.0	3
4045	4807.6375	3.2228	21.1	3
γ 15223	5128.7938	3.2259	21.7	3
C 1351	2968.6410	3.2509	21.7	2
1352	2968.6542	3.2641	28.1	1
4032	4780.6389	3.2687	31.1	2
4943	5483.7972	3.2705	24.5	2
4046	4807.6875	3.2728	41.6	1
3856	4692.7611	3.2854	52.6	1
4827	5395.9354	3.3033	35.7	3
4033	4780.6764	3.3062	45.4	1
3857	4692.7833	3.3076	58.1	1
4900	5439.8930	3.3136	43.2	1
4944	5483.8451	3.3184	+29.8	1
4302	5047.8812	0.0662	−56.4	2
4991	5517.7826	0.0697	61.8	2
3861	4692.9264	0.0702	41.2	1
5082	5578.6465	0.0836	62.1	1
3862	4692.9486	0.0924	54.6	2
4303	5047.9104	0.0954	67.7	1
4304	5047.9341	0.1191	47.9	2
γ 15319	5169.6493	0.1341	55.1	3
14349	4716.6834	0.1633	48.8	3
14403	4726.8486	0.1868	53.3	3
C 4440	5159.6292	0.2557	−39.6	3

For U Sagittae, Fetlaar[1] has made both uniform and darkened solutions depending on the long series of visual estimates by Nijland. His results give:

	Uniform	Darkened
Semi-duration of eclipse	0.257 days	0.281 days
Semi-duration of totality	0.037	0.033
Inclination	84°33′	90°

Using these values for the time of eclipse and for the inclination and assuming that the star rotates in the same period as that of its orbital revolution, we may determine the expected velocity corresponding to any given phase for certain values of the rotation at the limb. The four normal points of the observations during partial phases, both

[1] *Loc. cit.*

preceding and following totality, are plotted in Figure 3, the horizontal axis representing the velocity-curve of the brighter star determined from observations outside eclipse. The problem is to determine some mean value of the linear rotational velocity which will most nearly represent the observed normals. While there is perhaps some improvement in the quality of the spectral lines as the eclipse progresses, it is not marked and the spread of the measured velocities

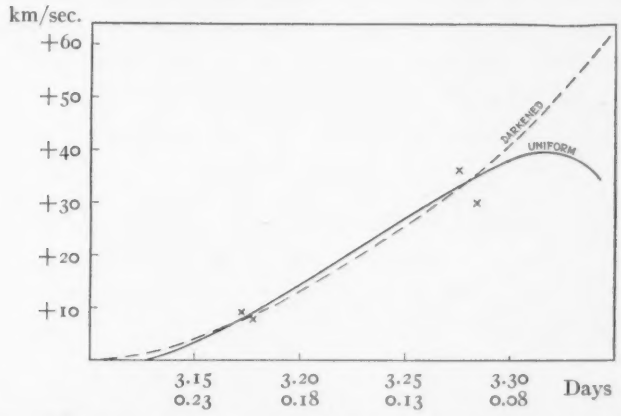


FIG. 3.—Rotation effect using Fetlaar's darkened and uniform solutions. The crosses are observed normal points. The observations after minimum are plotted in reverse direction. The curves represent the computed values.

is still large. The observations preceding minimum have somewhat greater total weight than those following and indicate a higher value of the rotation effect. Perhaps the simplest method of determining the amount of the rotation effect is the graphical solution used by Rossiter[1] in his study of β Lyrae. This method has been followed both for the uniform solution with an inclination of $84°33'$ and the completely darkened solution with central eclipse. For the darkened solution it was necessary to take into account not only the varying velocity, which increases directly with distance from the axis of rotation, but also the decrease in brightness from the center outward toward the limb in all directions. Following the procedure of Russell and Shapley[2] for darkened stars, we have drawn a disk representing the apparent surface of the brighter star and divided it into areas of

[1] *Astrophysical Journal*, **60**, 20, 1924. [2] *Ibid.*, **36**, 239, 1912.

equal light-intensity on the assumption that the light is reduced to zero at the limb in accordance with the law

$$J = J_0 \cos i ,$$

where i is the angle between the line of sight and the normal. The projection of the larger and fainter star was then drawn upon the figure as it would appear at certain successive phases of the partial eclipse. The uneclipsed areas of equal luminosity at stated distances from the axis of the star were counted, and the mean velocity of that portion of the star which was visible at the several phases was thus determined by mechanical quadrature for different values

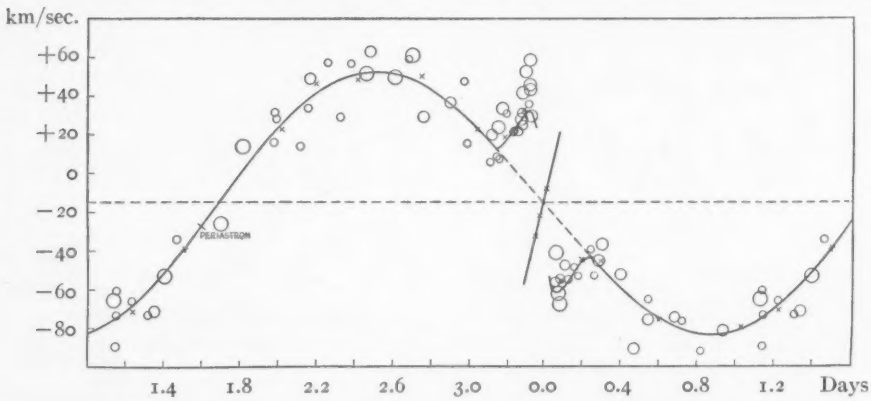


FIG. 4.—Velocity-curve of U Sagittae. Circles are single observations. Crosses are normals. The size of the circles varies inversely with the weight.

of the rotational velocity at the limb. The curves which best fit the weighted normal places are shown in Figure 3. The corresponding mean rotation effects are estimated to be 60 km/sec. for the uniform and 62 km/sec. for the darkened disks.

It is commonly believed that the uniform disk is more probable for stars of B type, and Fetlaar's uniform solution gives better agreement with the photometric observations than the darkened. The spectrographic observations in this case seem utterly inadequate for a decision between the two solutions. Although the forms of the curves are very different near the time of totality, they are much the same for those portions of the curves where the spectra are measurable. The observations of the B-type star cannot be pushed closer to minimum because the G-type star seems to dominate the spectrum

within thirty or forty minutes of totality, when the combined light is fainter than 8.6 mag. and its numerous lines render settings upon the diffuse hydrogen and helium lines of the early-type star uncertain.

The value of 60 km/sec. for the rotational velocity at the limb, which corresponds to the uniform solution, gives 2,790,000 km for the radius of the small, bright star. This result depends chiefly upon the spectrographic data and the period, although the value of the rotation effect also involves the duration of partial and total phases, the inclination of the orbit, and the relative sizes of the two stars as determined by the photometric solution. It agrees reasonably well with the radius computed by combining $a \sin i$ of the spectrographic orbit with the photometric data.

V. ABSOLUTE DIMENSIONS AND PHYSICAL PROPERTIES

Fetlaar's[1] solutions give the inclination of the orbit and the dimensions of the stars in terms of the relative orbit, which are as follows:

	Uniform	Darkened
r_1, radius of larger star	0.292	0.280
r_2, radius of small star	.175	.218
$\cos i$	0.095	0.000

By using these results with the spectrographic values of $a \sin i$ (3,155,000 km) and the ratio of the masses (3.3), the absolute dimensions may be determined as follows:

	Uniform		Darkened	
a_1, semi-major axis of primary orbit	3,169,000 km		3,155,000 km	
a_2, semi-major axis of secondary orbit	10,457,000		10,410,000	
a_1+a_2, semi-major axis of relative orbit	13,626,000		13,565,000	
r_1, radius of large star	3,979,000	(5.7⊙)	3,798,000	(5.4⊙)
r_2, radius of small star	2,385,000	(3.4⊙)	2,957,000	(4.2⊙)
ρ_1, density of large star	0.011⊙		0.013⊙	
ρ_2, density of small star	0.171⊙		0.090⊙	
M_1, visual absolute magnitude of large star	+1.8 mag.		+1.9 mag.	
M_2, visual absolute magnitude of small star	−0.4		−0.8	
j_1, surface brightness of large star	+0.8		+0.8	
j_2, surface brightness of small star	−2.6		−2.5	

[1] *Loc. cit.*

The absolute magnitudes are derived from the formula[1]

$$M=\frac{29,500}{T}-5\log r-0.08\,.$$

The temperatures (T) are based on the spectra and taken to be 5200° for the larger star and 12,000° for the smaller.

The values of the surface brightness, which are given with reference to the sun ($M\odot=4.85$), depend on the relationship

$$j=5\log r+M-4.85\,.$$

The absolute magnitudes and masses satisfy the mass-luminosity correlation of Eddington very closely. From the absolute and apparent magnitudes ($m=6.7$ and 9.4) the parallax is found to be 0".003.

The spectroscopic absolute magnitude of the faint star determined from the recent Mount Wilson reduction-curves is +1.4.

The data from the various sources show satisfactory agreement. Further observations, either spectrographic or photometric, would add to the accuracy of our knowledge of this interesting system; but it seems doubtful whether there would be very great changes in the conclusions presented here. The times of minimum should be observed during the next six years for further confirmation of the change in period.

Much of the computation of the solution for the elements of the orbit and the rotation effect was done by Mr. N. U. Mayall.

CARNEGIE INSTITUTION OF WASHINGTON
MOUNT WILSON OBSERVATORY
March 1930

[1] Russell, Dugan, and Stewart, *Astronomy*, p. 732, 1927.

THE RAYTON SHORT-FOCUS SPECTROGRAPHIC OBJECTIVE[1]

By MILTON L. HUMASON

ABSTRACT

Spectra of very faint stars and nebulae can be observed only with a camera objective of short focus. The California Institute of Technology has a lens of this type, later to be used with the 200-inch telescope, which is an eight-fold enlargement of a microscope objective, with a focal length of 32 mm and an aperture of 50 mm or a focal ratio of $F/0.6$.

Tests made at the 100-inch telescope of the Mount Wilson Observatory show excellent definition and a gain of 50 per cent in speed over the short-focus camera previously used. The spectrograph has a collimator of 24 inches focal length and two light flint prisms. The dispersion at λ 4350 is 418 A per millimeter. Ordinary stellar velocities cannot be measured, since the probable error is of the order of 50–100 km/sec.; but an uncertainty of this amount is not important when large displacements, such as those occurring in the spectra of faint extra-galactic nebulae, are measured.

A single nebula in the Ursa Major cluster has been observed in which the apparent velocity-shift is +11,500 km/sec. The exposure time required was forty-five hours.

The spectra of very faint objects are difficult to obtain on account of the small amount of light made available by even the largest telescopes. The only way in which they can be observed is to use a camera lens of very short focus, which reduces the spectrum to the smallest possible length and width, thereby cutting down the exposure time. Practically the greatest difficulty has been to obtain a short-focus lens which would give good definition.

The California Institute of Technology has secured a lens designed by Dr. W. B. Rayton, of the Bausch and Lomb Optical Company, which is later to be used in connection with the 200-inch telescope. It is an eightfold enlargement of a microscope objective, gives good definition, and has great speed. A technical description by Dr. Rayton will be published elsewhere in this *Journal*.

The Mount Wilson Observatory has tested this lens in a spectrograph mounted at the Cassegrain focus of the 100-inch telescope. The spectrograph includes a collimator of 24 inches focal length, two 60° light flint prisms, and the new Rayton objective, which has a focal length of 32 mm and an aperture of 50 mm or a ratio of $F/0.6$. The consequent reduction from slit to plate is nineteen times, and

[1] *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 400.

makes possible the use of a very wide slit without an appreciable loss of definition or widening of the lines. The instrument shops of the Mount Wilson Observatory are now building a new spectrograph in which this objective will be used. It will have the same optical parts as the one now in use, but the slit will be curved instead of straight, thereby eliminating most of the curvature of the lines, which is quite noticeable with the straight slit now in use. The dispersion is 418 A per millimeter at λ 4350, and the distance between the helium lines λ 3888 and λ 5015 is 2.67 mm. Helium has been used as the source for the comparison spectrum as the number and spacing of the lines in the photographic region have been found satisfactory for the dispersion.

The probable error of measures of line displacement is necessarily large, as large or larger than the ordinary stellar velocity. It is therefore not feasible to use this lens for measurements of stellar velocities, but it can be used successfully for photographing certain classes of very faint stars and nebulae which show very large line displacements and for which an accurate measurement of the displacements in any single object is not expected.

During the few months in which this camera lens has been in use it has been employed almost entirely in observing the faint and very remote extra-galactic nebulae, especially those which occur in clusters and have large apparent radial velocities. It has been found to be an ideal type of spectrograph objective for such objects. The wide slit can be used to great advantage, and although the probable error of the measured displacements amounts to 100 km/sec., the gain in speed becomes more important than the loss in accuracy. Since the very remote nebulae have apparent radial velocities of 2000 km/sec. or more, a probable error of this order is small as compared with the velocity displacements. The absorption lines or blends observed in the nebulae are more diffuse than in stellar spectra and are few in number. The strongest and most easily recognized are the H and K lines of calcium, which furnish a sure indication of the amount of the displacement and make possible the easy identification of the remaining lines, $H\delta$, the G (λ 4308) band, and $H\gamma$.

The following indicates other investigations for which this lens will be used: The mean velocity from a group of individual stars in

a cluster should give a very good value of the velocity of the cluster as a whole. The spectra of faint Cepheid variables may be studied. An accurate classification of the spectra of faint stars of large proper motion may be made, and, as most of these stars have spectra of late K or M types, a reliable value of the absolute magnitude may be obtained from the spectral classification alone. Photographs of the spectra of novae which occur in the Andromeda nebula and which seldom reach a photographic magnitude brighter than 16.0 at maximum light should be possible and would be very valuable. Spectroscopic records of the minimum phase of galactic novae may also be obtained. The features of the absorption measurable in stellar spectra vary with the type, and the probable error becomes smaller with the use of a narrow slit. The slit width most often used for stellar observations is 0.18 mm, and the probable error of the measurement from a single plate is 50 km/sec. For the nebulae a slit width of 0.20–0.60 mm has been used according to the brightness of the object observed.

In order to give some idea of the speed of the new 1.3-inch camera it may be compared with the one in use just previously. The old camera had a focal length of 3 inches and formed a part of the same spectrograph, having exactly the same optical parts except that only one of the 60° prisms was used. The dispersion was 3.21 mm between λ 3888 and λ 5015, as compared with 2.67 mm for the new camera and two prisms. The reduction from slit to plate was therefore eight times as against nineteen times for the new camera. No direct test of the two instruments has been made, but spectrograms are available of two nebulae in the same cluster, N.G.C. 1270 and 1273, which have approximately the same brightness. N.G.C. 1273 was photographed with the old 3-inch camera and an exposure of twenty hours; N.G.C. 1270 was taken with the new camera and exposed nine hours. The two spectrograms, enlarged fourteen times, are shown in Plate XI. It was difficult to enlarge these spectrograms from their original scale of about 3 mm to one suitable for reproduction, and the illustration does not show all of the added fine detail recorded by the new camera. The negatives show, however, that the spectrogram taken with the new camera is much superior in definition and density to that photographed with the old lens. One of the

striking features of this lens is the lack of coma, which has been a rather persistent difficulty with most other short-focus spectrographic objectives tried at this observatory. A further test of efficiency is the exposure time required to obtain a spectrogram of a star of photographic magnitude 12.0. With fair seeing this is ten minutes for the new camera as against thirty minutes for the 3-inch camera.

The color-curve of the objective is shown in Figure 1, where the points and the full-line curve represent the best focal setting for a given wave-length with a flat plate perpendicular to the optical axis. The crosses and the broken curve represent the best settings for the

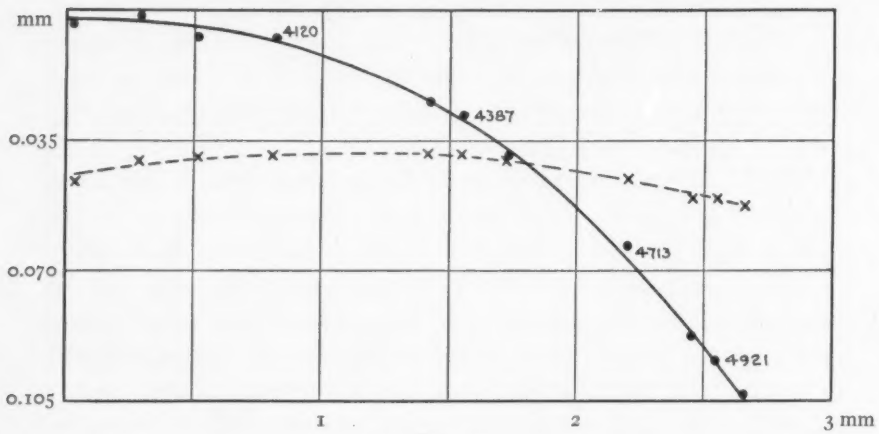
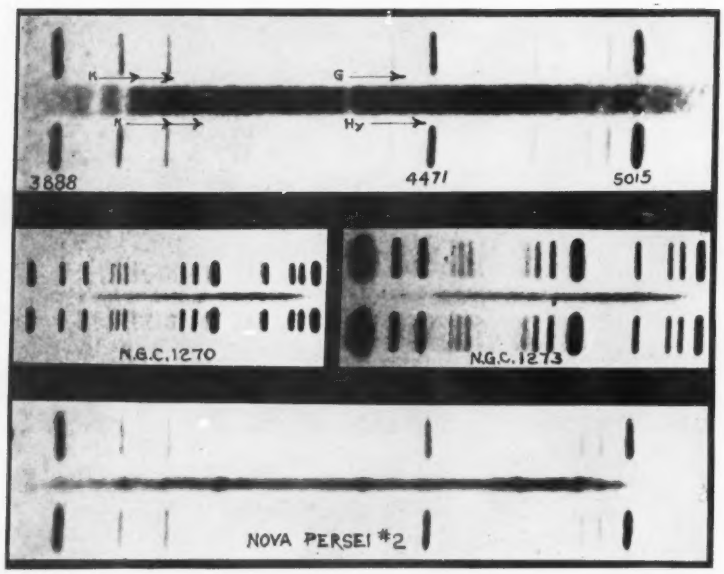


FIG. 1

same wave-lengths with a tilted, curved film. The vertical scale represents the focal settings and the horizontal scale the separation in millimeters of the different wave-lengths, a few of which have been indicated on the plot. The horizontal scale is fourteen times that of the vertical scale. The full-line curve shows the amount of tilt and the curvature which was required in the plate-holder. The tilt has been introduced by displacing the center of curvature of the curved plate in the plate-holder. The broken curve shows that the curvature used was insufficient, but the error is so small that no attempt will be made to correct it.

Eastman portrait films, which are comparable to the Eastman 40 plates in speed and grain, are used at the present time. The size required by the new plate-holder is 1-5/16×5/8 inches.

PLATE XI



EXAMPLES OF SPECTRA TAKEN WITH RØYTON CAMERA LENS 1A

a) Sky, with Comparison Lines of Helium
b) N.G.C. 1270 and N.C C. 1273
c) Nova Persei No. 2

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The design of the plate-holder was difficult on account of the very small clearance between the film and the lens cell, the focus being only 2 mm back of the rear surface of the lens. A plate-holder of the usual design was tried first, but proved unsatisfactory because the plate-base had to be backed away from the lens until the plate-holder was clamped on and the slide pulled. This arrangement has now been discarded and a plate-holder designed which clamps on to the plate-base, bringing the film into focus without further adjustment. This new plate-holder not only has a slide but carries a curved plate for bending a photographic film, which makes it possible to correct for the tilt and for the color-curve of the lens. The bottom part of the plate-holder carrying the slide consists of three sheets of metal 0.007 inch in thickness, each having a hole 8 mm in diameter. The middle sheet acts as the slide and moves between the outer two, the bottom or outside piece of metal being turned up and soldered to the sides of the plate-holder. When the three openings coincide, the film is exposed, and when the hole in the slide is pushed beyond the holes in the outer pieces, the plate-holder is closed.

When the camera is in focus and the plate-holder in place, a clearance of only 0.1 mm remains between the bottom of the plate-holder and the lens cell. This plate-holder is now in use and has been found very convenient and light-tight.

The spectra reproduced in Plate XI are as follows: At the top is a widened spectrogram of skylight taken with a narrow slit. The calcium lines H and K are seen, the G band, and $H\gamma$. The wave-lengths of three helium lines in the comparison spectrum have also been indicated. The absorption lines in the sky spectrum are the strongest appearing on spectra of very small dispersion, and are those most often measured in the spectra of nebulae. The tips of the arrows to the right show where these lines appear in the spectrum of N.G.C. 4860, a nebula in the Coma Berenices cluster having an apparent radial velocity of +7800 km/sec. Just below the sky spectrum are shown spectra of two nebulae in the Perseus cluster already mentioned, N.G.C. 1270 and N.G.C. 1273. The mean apparent radial velocity of the nebulae in this cluster is +5100 km/sec., and the arrow-tips to the left in the sky spectrum show where the H and K lines appear in these nebulae. A direct photograph showing a small

region of the cluster of spirals in Perseus is seen in Plate XII. The spectrum at the bottom of Plate XI is of Nova Persei No. 2, photographed in January, 1930, by F. G. Pease with four hours' exposure at the 100-inch telescope. This nova was at maximum in 1901, but was estimated as close to 13.0 when photographed by Pease. It has been included as an illustration of an emission spectrum. The seeing was very poor and the spectrum was widened by moving the star along the slit. Another plate of Nova Persei, not widened and taken during better seeing, was obtained by Pease with an exposure of thirty minutes. The emission lines shown are $H\delta$, $H\gamma$, 4686, and $H\beta$. The two large-scale spectrograms have been enlarged twenty-eight times, and the small-scale spectrograms fourteen times, from the original.

For very faint objects observable only with a small-scale camera the speed of our spectrograph has been doubled by the use of the Rayton lens. It is now possible to observe faint objects which have heretofore seemed hopeless; one of these is a very faint cluster of nebulae in Ursa Major described by W. Baade.[1] This is probably one of the most remote clusters known at present. Baade estimates its distance as one hundred and fifty million light-years, while Hubble from recent data put it at a distance of the order of eighty million light-years. A single observation of one nebula in this cluster has just been made, and the displacement is +11,500 km/sec. The distance obtained from the displacement by the use of Hubble's velocity-distance correlation, in which 500 km/sec. displacement to the red corresponds to a distance of one million parsecs, is of the order of twenty-three million parsecs, or seventy-five million light-years. This is approximately the same as the distance which Hubble obtains from observations of the total brightness of the nebulae involved in the cluster. The exposure time required to photograph the spectrum of this faint object was forty-five hours. Spectroscopic observations with any other short-focus camera would have required an exposure of one hundred hours or more.

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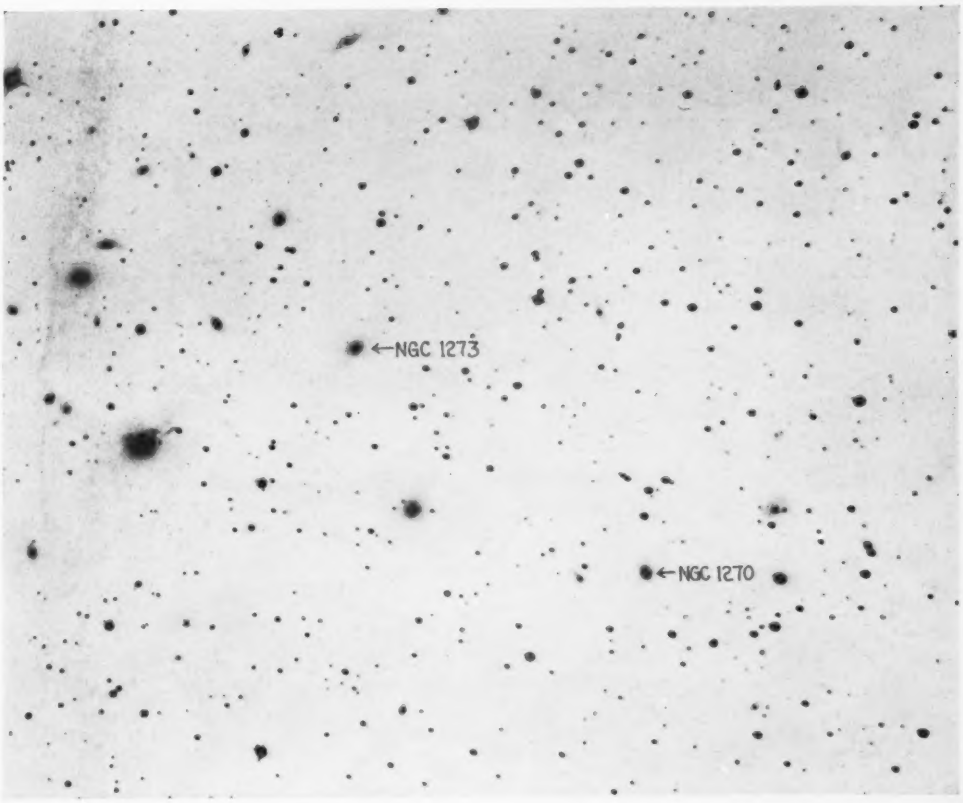
May 1930

[1] *Astronomische Nachrichten*, **233**, 65, 1928.

PLATE XII



DIRECT PHOTOGRAPH OF REGION OF CLUSTER OF SPIRALS IN PERSEUS, INCLUDING N.G.C. 1270 AND 1273

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INDEX TO VOLUME LXXI

SUBJECTS

PAGE

Absorption, Distinction between Scattering and. *John Q. Stewart* and *Serge A. Korff* 62

Al III and of Al II in Stellar Spectra, Identification of. *Otto Struve* . . 67

7ϵ Aurigae, Preliminary Results of Spectrographic Observations. *O. Struve* and *C. T. Elvey* 136

Band-Spectrum of He_2 in Stellar Spectra, Search for. *O. Struve* and *A. Christy* 277

Binaries, Spectroscopic Observation of Eclipsing. *Eugene W. Pike* . . 70

Chromosphere, Spectrum of. *S. A. Mitchell* 1

Energy-Curve of Sun-Spots, Spectral. *Edison Pettit* and *Seth P. Nicholson* 153

He_2 in Stellar Spectra, Search for Band-Spectrum of. *O. Struve* and *A. Christy* 277

Hydrogen Lines in Stellar Spectra, Contours of. *C. T. Elvey* . . . 191

Luminosity Distribution in Elliptical Nebulae. *Edwin Hubble* . . . 231

Lunar Radiation and Temperatures. *Edison Pettit* and *Seth P. Nicholson* 102

R Lyrae, Radial Velocity of. *Roscoe F. Sanford* 209

Magnitudes, Stellar Absolute, from Distributions of Angular and Linear Peculiar Velocities, Method for Determining Distribution of. *Gustaf Strömberg* 163

Magnitudes, Distribution of Absolute, among K and M Stars Brighter than the Sixth Apparent Magnitude as Determined from Peculiar Velocities. *Gustaf Strömberg* 175

Mg^+ 4481, Rotation of Stars and the Contours of. *C. T. Elvey* . . . 221

Nebulae, Luminosity Distribution in Elliptical. *Edwin Hubble* . . 231

Notice to Contributors 72

Phosphorus in Stellar Spectra. *Otto Struve* 150

Preparation of Abstracts 220

Radial Velocity of R Lyrae. *Roscoe F. Sanford* 209

Rayton Short-Focus Spectrograph. *Milton L. Humason* 351

Reviews:

M. Born and P. Jordan. *Elementare Quantenmechanik* (Carl Eckart) 283

Peter Doig. *An Outline of Stellar Astronomy* (A. S. Fairley) . . 219

Walter Gerlach. *Matter, Electricity, Energy* (B. P. Gerasimovič) . 217

Arthur Edward Ruark and Harold Clayton Urey. *Atoms, Molecules and Quanta* (Frank Hoyt) 218

PAGE

U Sagittae, Spectrographic Study of. *Alfred H. Joy* 336
Scattering and Absorption, Distinction between. *John Q. Stewart* and *Serge A. Korff* 62
Spectra, Contours of Hydrogen Lines in Stellar. *C. T. Elvey* . . . 191
Spectra, Identification of Al III and of Al II in Stellar. *Otto Struve* . . 67
Spectra, Phosphorus in Stellar. *Otto Struve* 150
Spectra of Several Long-Period Variable Stars, Behavior of Bright Lines in. *Paul W. Merrill* and *Cora G. Burwell* 285
Spectral Energy-Curve of Sun-Spots. *Edison Pettit* and *Seth B. Nicholson* 153
Spectrograph, Rayton Short-Focus. *Milton L. Humason* 351
Spectrographic Observations of 7ε Aurigae, Preliminary Results. *O. Struve* and *C. T. Elvey* 136
Spectrographic Study of U Sagittae. *Alfred H. Joy* 336
Spectrohelioscope and Its Work. *George E. Hale* 73
Spectroscopic Observation of Eclipsing Binaries. *Eugene W. Pike* . . 70
Spectrum of the Chromosphere. *S. A. Mitchell* 1
Stars and the Contours of Mg^+ 4481, Rotation of. *C. T. Elvey* . . 221
Sun-Spots, Spectral Energy-Curve of. *Edison Pettit* and *Seth B. Nicholson* 153
Variable Stars, Behavior of Bright Lines in Spectra of Several Long-period. *Paul W. Merrill* and *Cora G. Burwell.* 285
Velocities, Determining Distribution of Stellar Absolute Magnitudes from Distributions of Angular and Linear Peculiar. *Gustaf Strömberg* . . 163
Velocities, Distribution of Absolute Magnitudes among K and M Stars Brighter than Sixth Apparent Magnitude as Determined from Peculiar. *Gustaf Strömberg* 175

INDEX TO VOLUME LXXI

AUTHORS

PAGE

BURWELL, CORA G., and PAUL W. MERRILL. Behavior of Bright Lines in the Spectra of Several Long-Period Variable Stars 285

CHRISTY, A., and O. STRUVE. A Search for the Band Spectrum of He_2 in Stellar Spectra 277

ECKART, CARL. Review of: *Elementare Quantenmechanik*, M. Born and P. Jordan 283

ELVEY, C. T. Contours of Hydrogen Lines in Stellar Spectra 191

ELVEY, C. T. Rotation of Stars and the Contours of Mg^+ 4481 . . . 221

ELVEY, C. T., and O. STRUVE. Preliminary Results of Spectrographic Observations of 7 ϵ Aurigae 136

FAIRLEY, A. S. Review of: *An Outline of Stellar Astronomy*, Peter Doig 219

GERASIMOVIČ, B. P. Review of: *Matter, Electricity, Energy*, Walter Gerlach 217

HALE, GEORGE E. The Spectrohelioscope and Its Work 73

HOYT, FRANK. Review of: *Atoms, Molecules and Quanta*, Arthur Edward Ruark and Harold Clayton Urey 218

HUBBLE, EDWIN. Luminosity Distribution in Elliptical Nebulae . . . 231

HUMASON, MILTON L. The Rayton Short-Focus Spectrograph . . . 351

JOY, ALFRED H. A Spectrographic Study of U Sagittae 336

KORFF, SERGE A., and JOHN Q. STEWART. Distinction between Scattering and Absorption 62

MERRILL, PAUL W., and CORA G. BURWELL. Behavior of Bright Lines in the Spectra of Several Long-Period Variable Stars 285

MITCHELL, S. A. Spectrum of the Chromosphere 1

NICHOLSON, SETH B., and EDISON PETTIT. Lunar Radiation and Temperatures 102

NICHOLSON, SETH B., and EDISON PETTIT. Spectral Energy-Curve of Sun-Spots 153

PETTIT, EDISON, and SETH B. NICHOLSON. Lunar Radiation and Temperatures 102

PETTIT, EDISON, and SETH B. NICHOLSON. Spectral Energy-Curve of Sun-Spots 153

PIKE, EUGENE W. A Note on the Spectroscopic Observation of Eclipsing Binaries 70

SANFORD, ROSCOE F. On the Radial Velocity of R Lyrae 209

PAGE

STEWART, JOHN Q., and SERGE A. KORFF. Distinction between Scattering and Absorption 62

STRÖMBERG, GUSTAF. Distribution of Absolute Magnitudes among K and M Stars Brighter than the Sixth Apparent Magnitude as Determined from Peculiar Velocities 175

STRÖMBERG, GUSTAF. A Method for Determining the Distribution of Stellar Absolute Magnitudes from the Distributions of Angular and Linear Peculiar Velocities 163

STRUVE, OTTO. Identification of Al III and Al II in Stellar Spectra . 67

STRUVE, OTTO. Phosphorus in Stellar Spectra 150

STRUVE, O., and A. CHRISTY. A Search for the Band-Spectrum of He_2 in Stellar Spectra 277

STRUVE, O., and C. T. ELVEY. Preliminary Results of Spectrographic Observations of 7ε Aurigae 136

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